EXAM ACHARYA

GPSC - CIVIL

Solid Mechanics

"Education is the most Powerful Weapon which you can use to change the world."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

GPSC - CIVIL
Solid Mechanics

INDEX

CHAPTER – 1

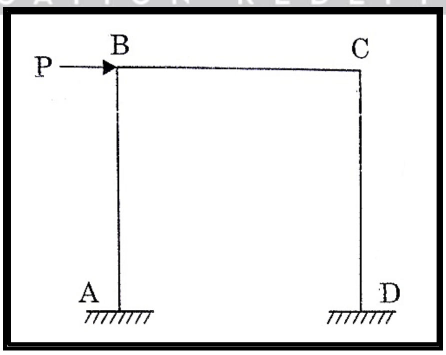
STRENGTH OF MATERIALS

INTRODUCTION

- Strength of Materials topic is to study the behaviour of a material when it is subjected to forces and moments.
- Each member of a structure is made up of certain materials, which could be a rigid material or a deformable material.

Rigid and Deformable Material

A rigid material is one which does not undergo any change in its geometry, size or shape. On the other hand a deformable material is the one in which change in size, shape or both will occur when it is subjected to a force/moment. The geometrical changes produced are called deformations and hence the name deformable material. All materials are actually deformable and the idea of rigid material is only a conceptual idealization



STRESSES AND STRAINS

- When we apply forces on solids, deformations are produced if the solid is restrained from motion either fully or partially.

Internal Force

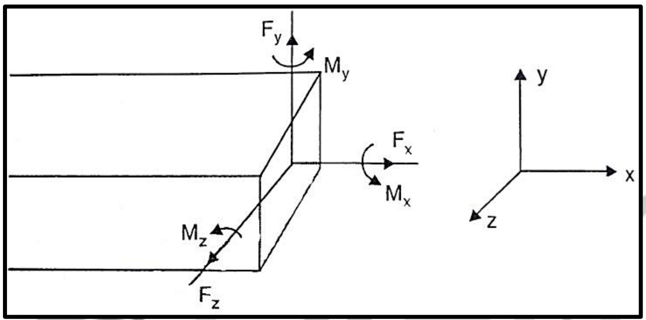
All external forces causes internal forces and at any section we have six internal forces

F_x = axial force

$\left.\begin{matrix} F_Y \\ F_Z \end{matrix}\right\}$ = Shear force

M_x = Twisting moment

$\left.\begin{matrix} M_Y \\ M_Z \end{matrix}\right\}$ = Bending moment



Hence, there are 4 types of internal forces

They are (i) Axial force (Normal stress)

(ii) Shear force (shear stress)

(iii) Bending moment (normal stress)

(iv) Twisting torsional moment (shear stress)

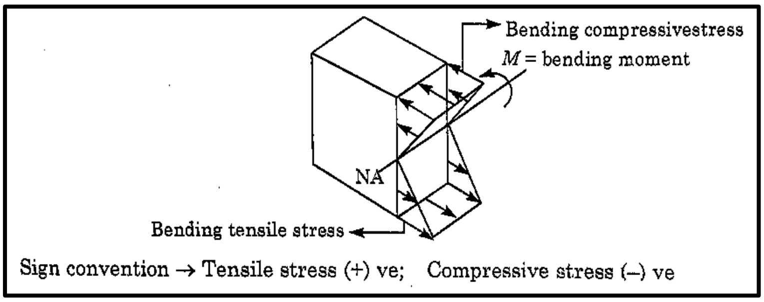
NORMAL STRESS

Normal stresses can be:

(a) Axial stress (b) Bearing stress (C) Bending stress

Bending Stress

Bending tension and compression produces normal stress.



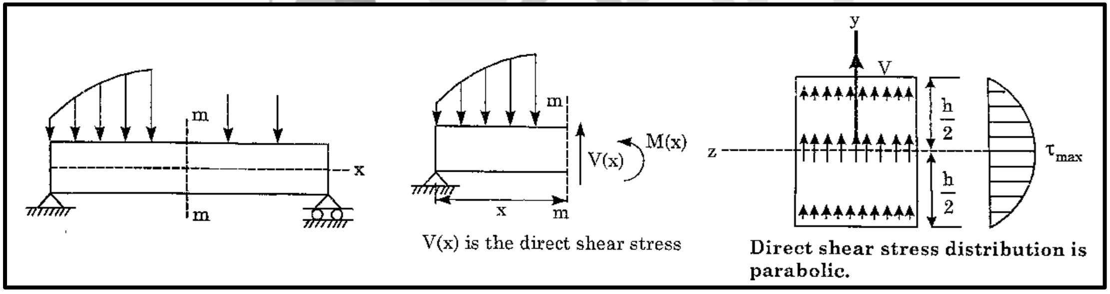
SHEARING STRESS

It is the stress acting in the plane of a section. The shearing stress could be

(a) Direct shear stress (b) Indirect shear stress

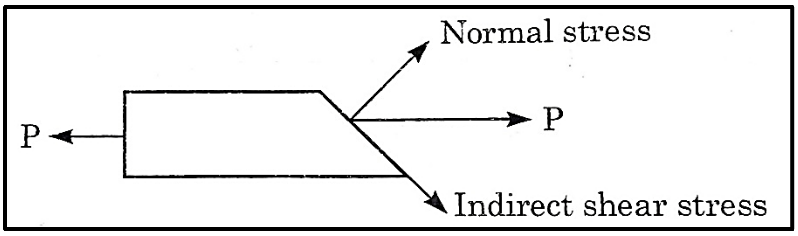
Direct Shear Stress

Shear stress is created due to direct action of forces in trying to cut through the material.

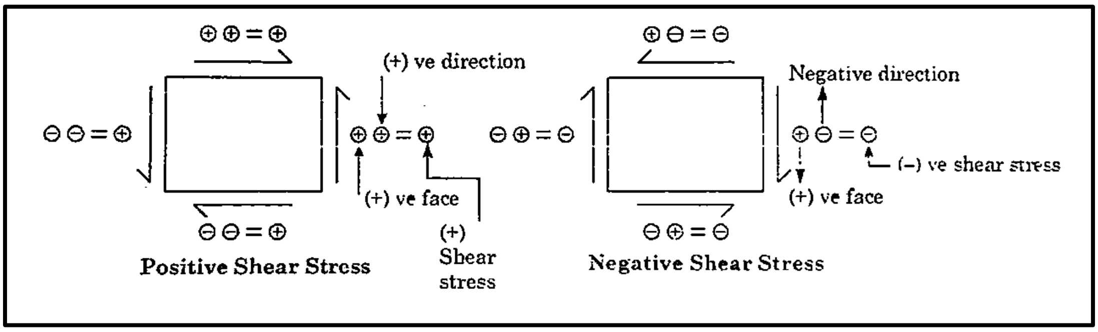


Indirect Shear Stress

Indirect shear stress arises due to (a) tension or compression (b) torsion. The figure below shows that shear stress arises in an indirect manner when members are subjected to tension/compression.

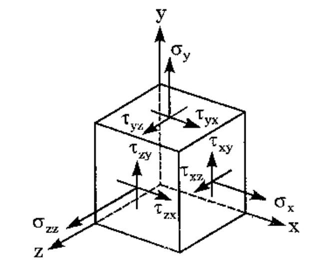


Thus



STRESS UNDER GENERAL LOADING CONDITIONS

Stress at any point under most general loading condition is as shown below in the figure.



Stress at any point (stress tensor)

τ_{xz} = shear stress on x-face in z-direction.

σ_{xx} = normal stress on plane normal to x face in x-direction

Stress is not a vector because its resultant cannot be obtained by parallelogram law of vector addition. It is a mathematical quantity called tensor.

Stress tensor is represented as:

$$\boldsymbol{\sigma} \text{ (stress tensor)} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Stress is a 2nd order tensor

Note
- Magnitude has only one dimension hence it is α 3° = zero-order tensor.
- Direction has three dimensions (x-direction, y-direction and z-direction) hence it is α 3^1 = lst order tensor. Stress has 9-dimensions (3^2= 2nd order tensor).

- At any point in 3D condition, we have 9-stress component

  3-Normal stress component (σ_{xx} , σ_{yy} , σ_{zz})

  6-Shear stress component (τ_{xy} , τ_{xz} , τ_{yx} , τ_{yz} , τ_{zx} , τ_{zy})

NORMAL STRAIN

Normal strain is defined as deformation per unit length.

$$\text{Normal strain } \varepsilon = \frac{\delta}{L}$$

Where, δ = Change in length and L= Actual length.

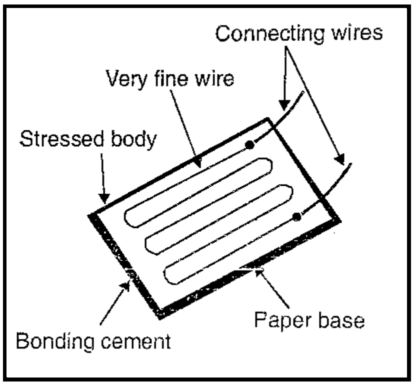
The above value gives only the average value of strain. The correct value of strain at any position is

$$\varepsilon = \frac{d\delta}{dL}$$

$d\delta$ = differential elongation (small elongation)

dL = differential length (small length)

- Strain is a fundamental behaviour of a material while stress is a derived concept.
- Strain has no unit.
- Normal strain is measured using extensometer.
- Normal strain on the surface of an element is measured using wire strain gauge. Figure shows that on a body a loop of wire is pasted and then it is stressed. Due to stress, strain occurs and length of the wire as well as diameter of wire changes.



Note

➢ Because of increase in length of wire and decrease in diameter, electrical resistivity of wire changes and if current is passed through the wire, its value changes on straining. By correlating the change in current with change in length, strain can be measured.

Test Series Available..

Total weekly test	**: 35**
Total mid subject test	**: 16**
Total full length test	**: 13**
Mock test	**: 16**
Total test	**: 80**

Some Important Points About Stress-Strain Curve of Mild Steel

Region OA

- From O to A stress is proportional to strain.

  i.e., $\frac{\text{stress}}{\text{strain}}$ = constant = E = modulus of elasticity = slope of OA

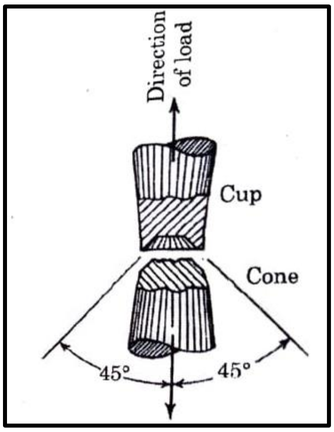
- Strains are infinitesimal.
- Volume of specimen increases due to tension.

Region AD

- Beyond A, strain increases more rapidly as compared to stress.
- If specimen is unloaded at point B, the unloading curve will be B-A-O.
- Upper yield point corresponds to the load reached just before yield starts.
- Lower yield point corresponds to the load required to maintain yield.
- Position of upper yield point is affected by speed of testing and by the shape of cross-section.
- Since upper yield point is transient, the lower yield point should be used to determine the yield strength of material. Lower yield point is considered to be the true characteristic of material.
- Volume of specimen increases.

Region DE

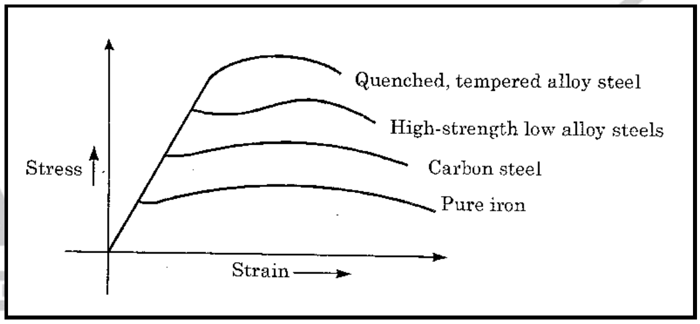
- Once yield stress is reached, the specimen undergoes large deformation with a relatively small increase in applied load. This is called plastic deformation.
- This deformation is caused by slippage of the material along oblique surfaces and is due therefore primarily to shearing stresses. The deformations are permanent. Volume of specimen does not change.
- Note that under uniaxial tensile stress, max shear stress is at 45° (oblique) angle to normal stress.



Test specimen at the time of tensile failure (Cup-Cone Failure)

STRESS STRAIN CURVE FOR OTHER MATERIALS

Some of the physical properties of structural metals, such as strength, ductility and corrosion resistance, can be greatly affected by alloying, heat treatment and the manufacturing processes used.



- For these 4-different grades of steel, yield stress, ultimate stress and fracture strain (ductility) differ greatly.
- All of them possess the same modulus of elasticity [i.e. stiffness with linear range is same].
- Note that, as yield strength increases, ductility falls.
- Mild steel has low carbon content as compared to HYSD. As carbon content increases

  (i) Ductility decreases

  (ii) Ultimate strength of steel increases

  (iii) Corrosion resistance reduces.

Various ductile materials are Al, Cu, Mg, Pb, Nickle, Boron, Bronze, Nylon, Teflon etc.

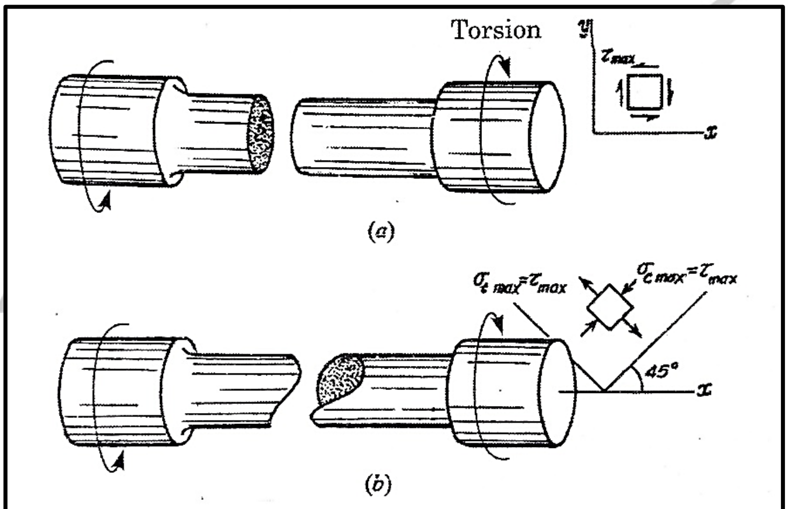
TYPE OF FAILURE IN TORSION TEST

- Brittle material fails at 45°
- Ductile material fails at 90°

  [Because brittle material fails due to normal stress which is max at 45° to the axis in case of torsion, while in ductile material failure is due to shear which, in case of torsion, occurs at 90° to the axis.]

However, in Tension test

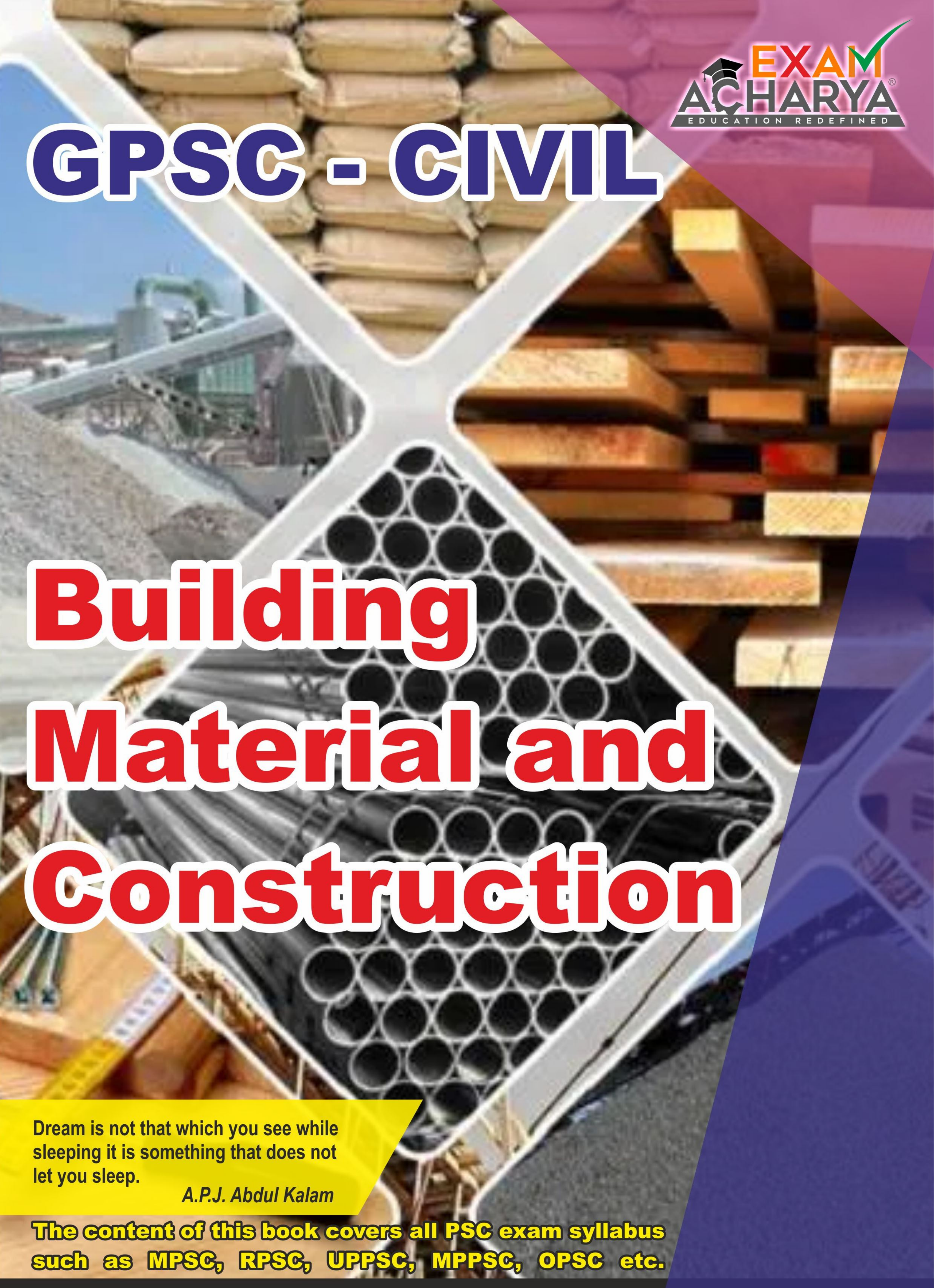
- Brittle material fails at 90°
- Ductile material fails at 45°



Fracture of (a) mild steel and (b) cast iron in torsion.

Brittle Materials in Compression

- For most brittle material, the ultimate strength in compression is much larger than ultimate strength in tension.
- This is due to the presence of flaws, such as microscopic cracks as cavities, which tend to weaken the materials in tension, while not appreciably affecting its resistance to compressive failure.



EXAM ACHARYA
EDUCATION REDEFINED

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Building Material and Construction

Dream is not that which you see while sleeping it is something that does not let you sleep.

A.P.J. Abdul Kalam

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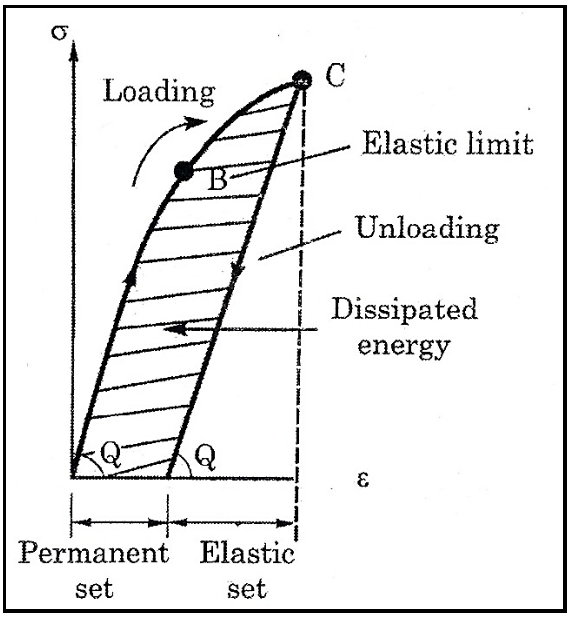


Plasticity

The characteristic of a material by which it undergoes inelastic strain beyond the strain at the elastic limit is known as plasticity.

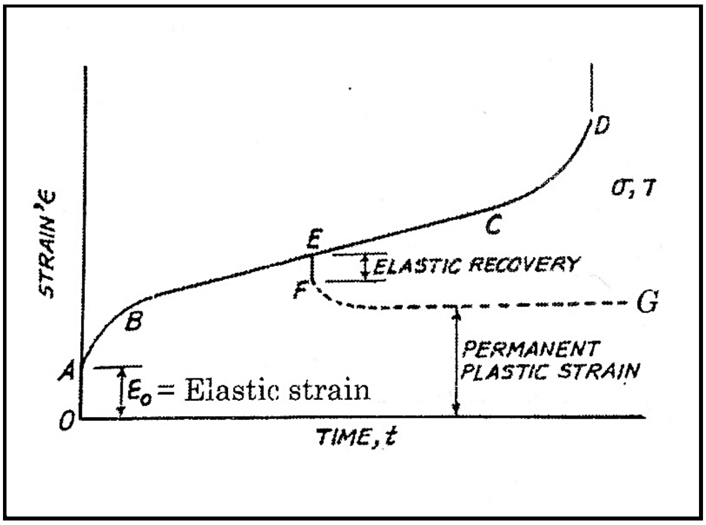
Ductility

- It is a measure of the amount by which a material can be drawn out in tension before it fractures. Ductility measurement can be done using tension testing.
- Ductility measurement can also be done using bend test in which sample of the material is bent through an angle and observed for appearance of crack. The angle at which cracking occurs gives a measure of ductility.

Malleability

Whereas ductility is the ability of a material to be drawn out in tension, malleability is the ability of a material to be deformed or spread in different directions. This is usually caused by compressive forces during rolling, pressing and hammering action. Copper is both ductile and malleable but lead is extremely malleable but not ductile and soon fractures under tension.

- Creep is usually more important at high temperature and higher stresses.



- It depends on temperature level, stress level, time, type of loading (static or dynamic)
- Generally, effect of creep becomes noticeable at approx. 30% of melting point (in degree kelvin) for metals.
- Moderate creep in concrete is sometimes welcomed because, it relieves tensile stress that might otherwise lead to cracking.

  OA → elastic strain

  BD → creep region. If member is unloaded at point E, the strain will follow path EFG.
- Creep strain is not 100% recoverable.

Relaxation

- The decrease in stress in steel as a result of creep within steel under prolonged strain is called relaxation.
- If bar is stretched to σ_0 stress in time t_0 and thereafter left to bear that stress, then the stress will go on reducing and ultimately becomes constant.

- For non-ferrous metal stress at failure continues to decrease. Hence, we define fatigue limit as the stress corresponding to failure after a specified number of loading cycles.

- Various factors that affect fatigue failure are stress concentration (i.e., due to Notch), corrosion, residual stress, size of specimen, surface finish etc.

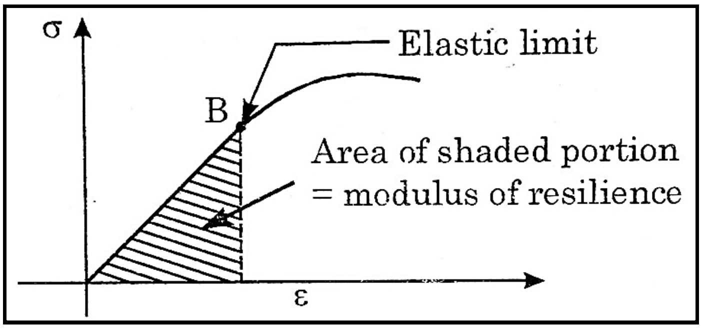
Resilience

- Stress concentration (↑) - fatigue strength (↓)
- Corrosion (↑) - fatigue strength (↓)
- Residual stress (↑) - fatigue strength (↓)
- Size (↑) - fatigue strength (↓)
- Polished surface - fatigue strength (↑)
- Temperature (↓) - fatigue strength (↑)

Definition of Resilience

- It is the property of a material to absorb energy when it is deformed elastically and then, upon unloading to have this energy recovered. Hence greater the resilience more desirable is the material for spring action.

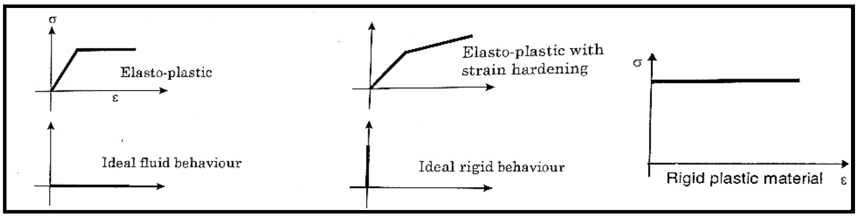
- The area under stress strain curve with in elastic limit is called modulus of resilience.



- For a linearly elastic material strain energy stored per unit volume

$$= \frac{1}{2}\sigma_y \cdot \frac{\sigma_y}{E}$$

APPROXIMATE STRESS-STRAIN CURVES



HOOKE'S LAW

Stress is proportional to strain (with in proportional limit).

$$\boldsymbol{\sigma = E\,.\,\varepsilon}$$

σ → Stress, E → Modulus of elasticity, ε → Strain

Hooke's law is vaild for homogeneous, isotropic and linearly elastic material.

DEFORMATION OF MEMBER UNDER AXIAL LOAD

Case 1: Bar of Uniform Section

1. If, $\sigma = \frac{P}{A}$, does not exceed proportional limit then

   Stress (σ) = Modulus of elasticity × Strain

   $$\sigma = E \cdot \varepsilon$$

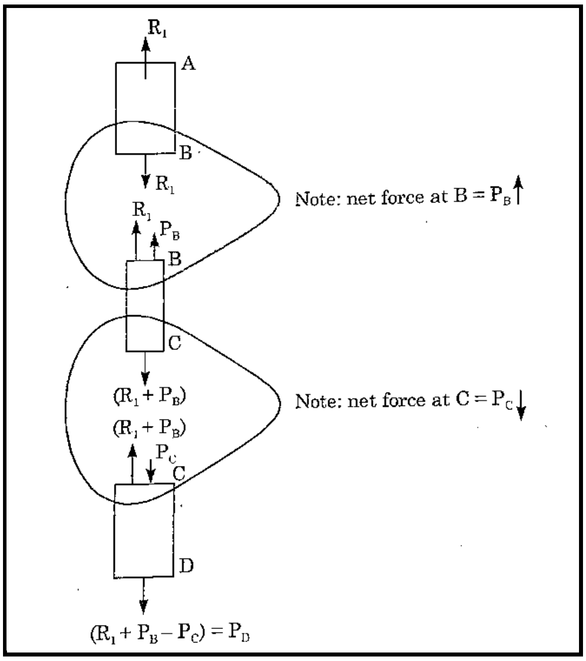
$$\Rightarrow \quad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

But $\varepsilon = \frac{\delta}{L}$

$$\Rightarrow \quad \frac{\delta}{L} = \frac{P}{AE}$$

$$\Rightarrow \quad \delta = \frac{PL}{AE}$$

For three bars as shown above forces in the bars will be calculated as follows:



For equilibrium of CD $R_1 + P_3 - P_c = P_D$ $\Rightarrow$ $R_1 = P_D + P_c - P_B$

Force in AB = N_1 $N_1 = P_D + P_C - P_B$

Force in BC $N_2 = R_1 + P_B = P_D + P_C$

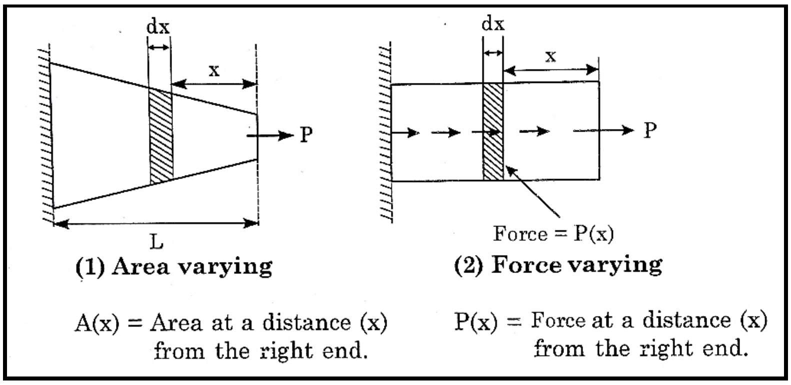
Force in CD $N_3 = P_D$

Total elongation of the bars = elongation of AB + elongation of BC + elongation of CD

$$= \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= \frac{N_1 l_1}{A_1 E_1} + \frac{N_2 l_2}{A_2 E_2} + \frac{N_3 l_3}{A_3 E_3}$$

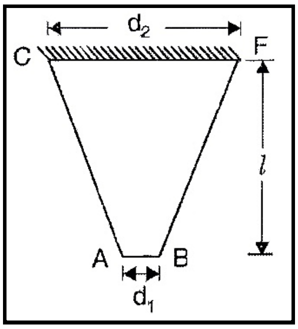
Case III: Bars of Varying X-Section or Bars Carrying Varying Forces



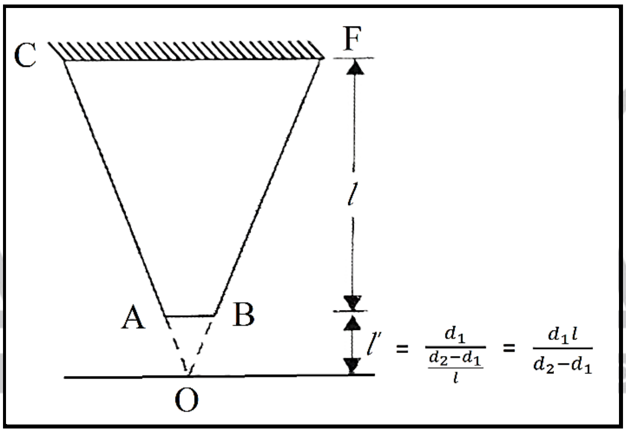
Example

Unit weight $=\lambda$ Modulus of elasticity = E

Find elongation due to self-weight, the bar has circular cross-section.



Solution



Elongation of ABCF due to self-weight = Elongation of OCF due to self-weight - Elongation of OAB due to self-weight

Elongation of ABCF due to weight of OAB $= \frac{\lambda(l+l')}{6E} - \frac{\lambda l'^2}{6E} - \frac{\left[\frac{1}{3}\pi\left(\frac{d_1}{2}\right)^2 l' x\lambda\right] l}{\frac{\pi d_1 d_2}{4} E}$

$$\delta = \frac{\lambda(l+l')^2}{6E} - \frac{\lambda l'^2}{6E} - \frac{\lambda l l'}{3E} \cdot \frac{d_1}{d_2}$$

$$= \frac{\lambda l^2}{6E}\left[\left(1+\frac{l'}{l}\right)^2 - \left(\frac{l'}{l}\right)^2 - \frac{2l'}{l} x \frac{d_1}{d_2}\right]$$

$$= \frac{\lambda l^2}{6E} + \frac{\lambda l^2}{6E}\left[\frac{2l'}{l}\right]\left[1 - \frac{d_1}{d_2}\right]$$

New Batches are going to start.....

Contact:

7622050066

Test Series Available..

Total weekly test	**: 35**
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Total test	**: 80**

Area of AB = A_1

Area of bar C = Area of bar C' = A_2

E_1 = modulus of elasticity of AB

E_2 = modulus of elasticity of C and C'

If a force 'P' is applied to AB, force on C and C' will be $\frac{P}{2}$ each

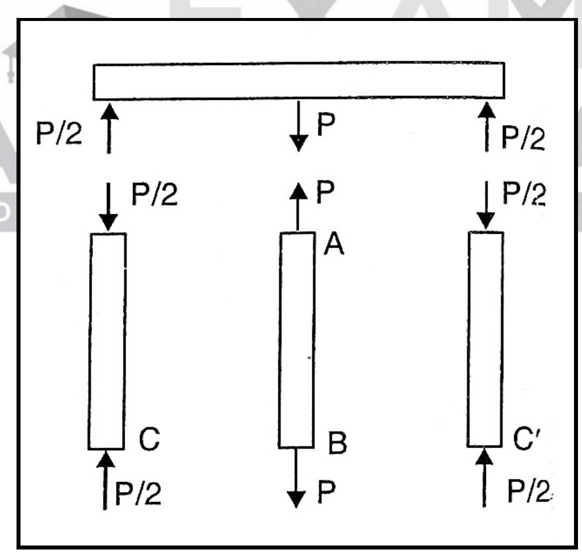
C and C' will be compressed by $\frac{(P/2)L}{A_2E_2} = \delta_A$

- Thus AB will have rigid body movement of δ_A.

Elongation of bar AB $\delta_{AB} = \frac{PL}{A_1E_1}$

Note that rigid body movement will not have any effect on elongation of a body.

- Point B will move down by $(\delta_A + \delta_{AB})$



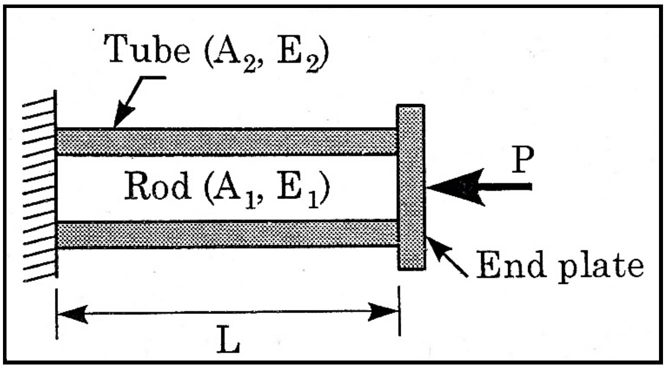
Example

A long rectangular copper bar under a tensile load 'P' hangs from a pin that is supported by two steel posts. The copper bar has a length of 2.0 m, x-section area = 4800mm^2, E_{copper} = 120 GPa. For each steel post, height = 0.5 m, x-section area = 4500mm^2 and E_{Steel} = 200 GPa.

$$\Rightarrow \frac{\delta_{Bmax}}{P_{max}} = \frac{\delta_B}{P}$$

$$\Rightarrow P_{max} = \frac{\delta_{Bmax} \times P}{\delta_B} = \frac{180 \times 1.0}{0.675} = 266.67 \text{ kN}$$

COMPOSITE BARS



We have to find forces in two member 1 and 2. Let force in the two members be P_1 and P_2

$$\Rightarrow \quad P = P_1 + P_2 \text{ ------------------(i)}$$

The other equation needed to find out P_1 and P_2 is obtained from compatibility condition. The compatibility condition is that change in length of the two members will be same.

$$\Rightarrow \quad \delta_1 = \frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2} = \delta_2 \text{------------------(ii)}$$

By solving (i) and (ii) P_1 and P_2 can be found out

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2}$$

$$P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$

PROBLEM INVOLVING TEMPERATURE CHANGE

If a body is free and temperature is raised then elongation will be $L\alpha\,\Delta T$, where 'α'is the coefficient of thermal expansion and ΔT= change in temperature of bar. The large is the value of 'α' the more sensitive is the material to temperature changes.

From compatibility $\delta_T = \delta_R = 0$

$$\Rightarrow \quad L\alpha\Delta T = \frac{RL}{AE}$$

$$\Rightarrow \quad \frac{R}{A} = Stress = E\alpha\Delta T$$

Note

- If temperature is increased and member is restrained, then, force produced is compressive. However, if temperature is decreased, the force produced is tensile.

TEMPERATURE CHANGE WITH YIELDING SUPPORT

If the temperature change in a restrained bar and support yields by amount δ.

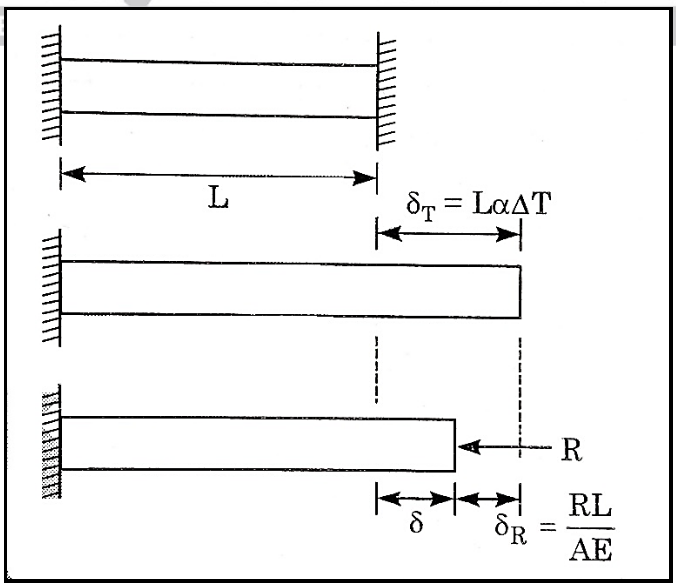
Then, $\delta_T - \delta_R = \delta$ = compatibility condition

$$L\alpha\Delta T - \frac{RL}{AE} = \delta$$

$$R = [L\alpha\Delta T - \delta]\ \frac{EA}{L}$$

$$\textbf{Stress} = \frac{R}{A} = \frac{E}{L}\,[L\alpha\Delta T - \delta]$$

$$\textbf{Stress} = \frac{E(L\alpha\Delta T - \delta)}{L}$$



Note

- Stress = $[\frac{\text{(Expansion prevented)}}{L}]$ E

As the ends of bar are free, net force on the composite bar even after temperature fall must be zero. Thus, compressive force in bar 2 must be equal to tensile force in bar 1.

$$\boldsymbol{\sigma_1 A_1 = \sigma_2 A_2}$$

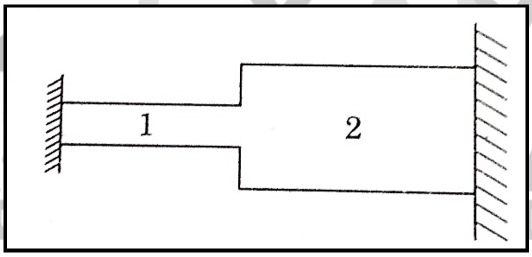
also, $\left(\frac{\sigma_1}{E_1}+\frac{\sigma_2}{E_2}\right)L = L(\alpha_1 - \alpha_2)\Delta T$

$$\Rightarrow \quad \boldsymbol{\frac{\sigma_1}{E}+\frac{\sigma_2}{E} = (\alpha_1 - \alpha_2)\Delta T}$$

Note
(a) Temperature rise ↑→ The material for which α ↑→ compression
(b) Temperature fall ↓→ The material for which α ↑→ tension

SERIES CONNECTION WITH END SUPPORTED

In this care expansion/contraction will be prevented by the supports hence compressive/tensile force will be exerted in both components 1 and 2.



If temperature is increased free extension = $L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

But this expansion has to be restrained. Hence compression develops in bars.

$$\frac{\sigma_1 L_1}{E_1}+\frac{\sigma_2 L_2}{E_2} = \text{compression}$$

$$\boldsymbol{\frac{\sigma_1 L_1}{E_1}+\frac{\sigma_2 L_2}{E_2} = L_1\alpha_1\Delta T + L_2\alpha_2\Delta T}\text{------------}\textbf{(i)}$$

σ_1 and σ_2 are compressive stress.

GPSC - CIVIL

Construction, Planning and Management

"All Birds find shelter during a rain.
But Eagle avoids rain by flying above the Clouds."

A.P.J. Abdul Kalam

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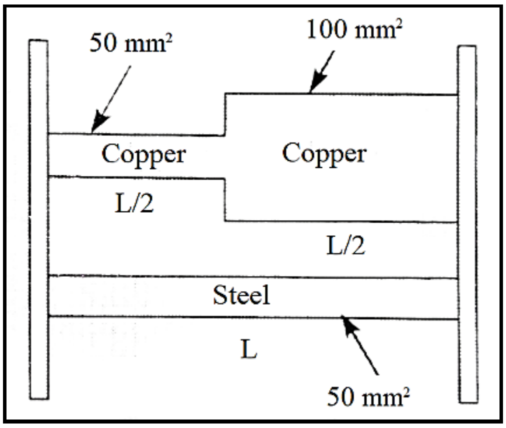
$E_s = 2 \times 10^5\ N/mm^2$, $\alpha_s = 1.2 \times 10^{-5}/°C$

$E_c = 10^5\ N/mm^2$, $\alpha_c = 1.2 \times 10^{-5}/°C$



Solution

Free expansion of copper = $\frac{L}{2}[\alpha_c \Delta T + \alpha_c \Delta T] = L\alpha_c \Delta T$

Free expansion of steel = $L\alpha_s \Delta T$

Since, $L\alpha_S \Delta T < L\alpha_C \Delta T$ hence, C_u will come under compression and steel will come under tension. Force in steel and copper will be same

$$\Rightarrow \quad \mathbf{P_c = P_S}$$

Compression of copper + extension of steel $= \delta = L(\alpha_c - \alpha_s)\Delta T$

$$\frac{\sigma_s L}{E_s} + \frac{\sigma_{c_1} L}{2E_c} + \frac{\sigma_{c_2} L}{2E_c} = L(\alpha_c - \alpha_s)\Delta T$$

$$\frac{P_s L}{A_s E_s} + \frac{P_c L}{2 x A_{c_1} E_c} + \frac{P_c L}{2 \times A_{c_2} E_c} = L(\alpha_c - \alpha_s)\Delta T$$

$$\text{Let, } P_S = P_c = P$$

$$A = A_S = A_{c_1} = \frac{A_{c_2}}{2}$$

$$\frac{PL}{AE_s} + \frac{PL}{2AE_c} + \frac{PL}{2 \times 2AE_c} = L(\alpha_c - \alpha_s)\Delta T$$

$$\varepsilon_z = \frac{-\mu\sigma_x}{E}$$

Note

- Volume of rod does not remain unchanged as a result of combined effect of elongation and transverse contractions.
- Dilatant material –Materials with μ > 0.5 undergoes increase in volume when compressed. Such materials are called dilatant
- Polymer foams expand laterally when stretched. Thus axial and lateral strains have same sign. Hence Poisson's ratio is negative.

- **For engineering materials, the value of Poisson's ratio ranges between 0.0 to 0.50**

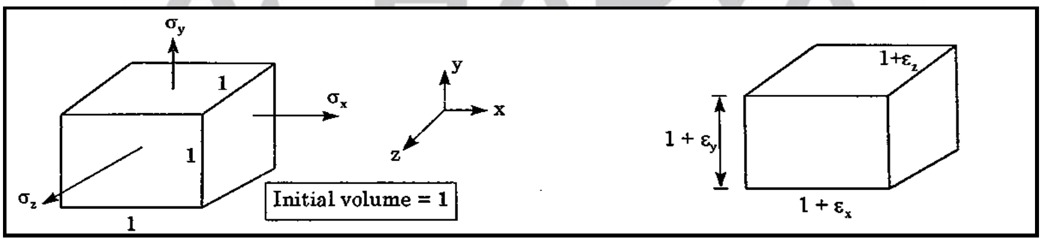
μ_{cork} = 0.0 $\qquad$ $\mu_{\text{perfectly elastic rubber}}$ = 0.5

μ_{concrete} = 0.1- 0.2 $\qquad$ μ_{metals} = 0.25 - 0.4

$\mu_{\text{Aluminium}}$ = 0.33 $\qquad$ $\mu_{\text{cast iron}}$ = 0.2 - 0.3

μ_{Steel} = 0.27 – 0.3 $\qquad$ μ_{rubber} = 0.45 – 0.5

DILATATION; BULK MODULUS



Final volume $= (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$

$$= (1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x)$$

Final volume $= (1+\varepsilon_x+\varepsilon_y+\varepsilon_z)$ [After neglecting product of strains, being small].

$$\Rightarrow \text{Change in volume} = \varepsilon_x+\varepsilon_y+\varepsilon_z$$

$$\Rightarrow \text{Volumetric strain } \varepsilon_v = \left(\frac{\Delta V}{V}\right) = \frac{(\varepsilon_x+\varepsilon_y+\varepsilon_z)}{1}$$

$$\Rightarrow \varepsilon_v \text{ is negative}$$

$$\Rightarrow \frac{E}{3(1-2\mu)} > 0$$

$$\Rightarrow \boldsymbol{\mu} < \frac{\mathbf{1}}{\mathbf{2}}$$

For all engineering material μ is (+) ve

$$\text{Hence } \mathbf{0} < \boldsymbol{\mu} < \frac{\mathbf{1}}{\mathbf{2}}$$

Note

- Equation 'A' shows that stretching of material in one direction i.e., due to σ_x ($\sigma_y = \sigma_z = 0$) will lead to increase in volume.
- During plastic deformation, volume of specimen remains constant.
- If $\mu = 0.5, \varepsilon_v = 0$ i.e., no volume change in Rubber.

Example

Determine the percentage change in volume of a steel bar 40mm square in section and 1 m long when subjected to an axial compressive load of 15 kN.

What change in volume would a 100 mm cube of steel suffer at a depth of 4 km in sea water?

Take E = 2 x 10^5 N/mm^2 and G = 0.81 x10^5 N/mm^2, $\gamma_{\text{sea water}}$ = 10080 N/m^3

Solution

(a) Volume of bar, $V = b^2.L$

$$\therefore \quad \frac{\delta V}{V} = 2\frac{\delta b}{b} + \frac{\delta L}{L} = 2e_b + e_L$$

$$\text{or} \quad \frac{\delta V}{V} = -\frac{2p\mu}{E} + \frac{p}{E} = \frac{p}{E}(1 - 2\mu) \text{----------(1)}$$

Now, $E = 2G(1 + \mu)$

$$\therefore \quad \mu = \frac{E}{2G} - 1 = \frac{2 \text{ x } 10^5}{2 \text{ x } 0.81 \text{ x } 10^5} - 1 = 0.2346$$

$\tau_{yz} = \tau_{zy}$

Hence no. of stress elements at a point to describe general stress situation are

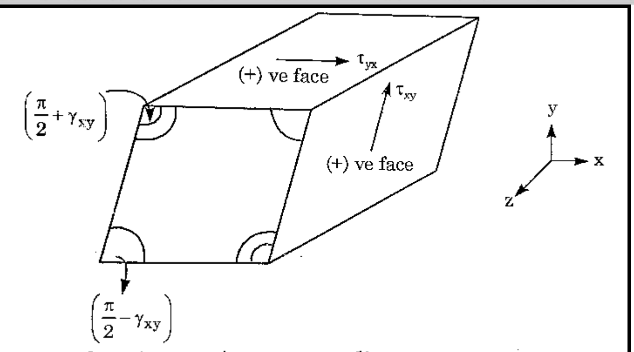
$\sigma_x, \sigma_y, \sigma_z, \Rightarrow$ 3 normal stresses

$\tau_{xy}, \tau_{yz}, \tau_{zx} \Rightarrow$ 3 shear stresses

- For homogeneous isotropic materials, shear stresses have no direct effect on the normal strain and as long as all the deformations involved remains small, they will not affect the derivation and validity of eq. 'α'i.e. $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$. Thus, equation of generalised Hooke's law derived earlier will still be valid even if shear stresses are acting.

- However, shear stresses will tend to deform the cubic element of material into an oblique parallelepiped. Deformation without change in volume is called distortion. If only shearing stresses are acting, then volume of the specimen does not change.

  If we consider only τ_{xy} andτ_{yx}, then the deformed shape will be as shown in the figure.



γ_{xy} = shearing strain corresponding to x and y direction

- When angle between two (+) ve faces as described earlier reduces, like from $\frac{\pi}{2}$ to $\left(\frac{\pi}{2} - \gamma_{xy}\right)$, γ_{xy} is (+) ve otherwise negative.

- Shear stress (τ_{xy}) and shear strain (γ_{xy}) curve is obtained from Torsion Test.

New Batches are going to start.....

Contact:

7622050066

Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\mu}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ \frac{-\mu}{E} & \frac{1}{E} & -\frac{\mu}{E} & 0 & 0 & 0 \\ \frac{-\mu}{E} & \frac{-\mu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

RELATION BETWEEN ELASTIC CONSTANTS

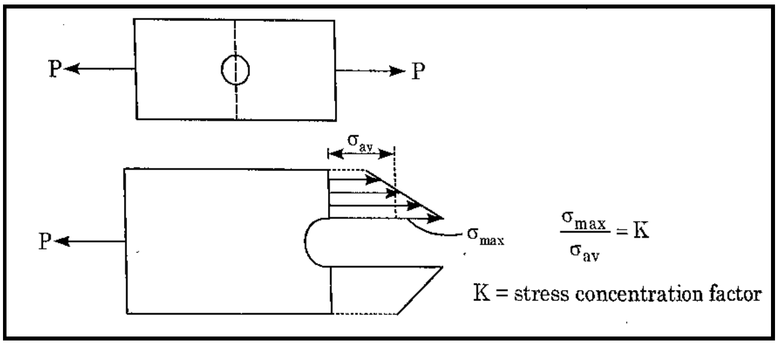
$$G = \frac{E}{2(1+\mu)} \qquad k = \frac{E}{3(1+2\mu)} \qquad E = \frac{9\,KG}{3\,K+G} \qquad \mu = \frac{3\,K-2\,G}{6\,K+2\,G}$$

If E and G are known, μ can be calculated. Hence for homogeneous and isotropic material there are only two independent and distinct elastic constants. (E and G, or E and μ, or G and μ).

- For isotropic material, normal strain does not depend on shear strain.
- For orthotropic material (like one laminate placed over other, eg. wood), Normal strain doesn't depend on shear strain. But Poisson's ratio and E are different in different direction.

Thus,

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{xz}}{E_x} & \frac{-\nu_{yx}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$



When load is increased such that

$$\sigma_{max} = \sigma_y \text{ (yield stress), then at that point}$$

$$P = P_y \text{ (i.e., load corresponding to 1st yield)}$$

$$\Rightarrow P_y = \sigma_{av} A = \frac{\sigma_y}{K} \cdot A$$

$$\mathbf{P_y = \frac{\sigma_y}{K} \cdot A}$$

When complete section has yielded $P = P_u$ = ultimate load

$$\mathbf{P_u = \sigma_y A}$$

$$\Rightarrow \frac{P_u}{P_y} = \frac{\sigma_y A}{\sigma_y A/K} = K$$

$$\mathbf{K = \frac{P_u}{P_y}} = \text{Stress concentration factor.}$$

PLASTIC DEFORMATION

When the yield stress in a material is exceeded, plastic flow occurs. To get a considerable insight into the plastic behaviour of the material an idealised curve is studied.

CLEAR YOUR CONCEPT

Qu1 **Match List – I with List – II and select the correct answer using the codes given below the lists:**

List-I	List-II
A. Ductility	1. Failure without warning
B. Brittleness	2. Drawn permanently over great changes of shape without rupture
C. Tenacity	3. Absorption of energy at high stress without rupture
D. Toughness	4. High tensile strength

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	1	2	3	4
(c)	2	3	4	1
(d)	2	1	4	3

Qu2 **Match List-I (Material) with List-II (Characteristic) and select the correct answer using the codes given below the lists:**

List-I	List-II
A. Inelastic material	1. No plastic zone
B. Rigid plastic material	2. Large plastic zone
C. Ductile material	3. Strain is not recovered after unloading
D. Brittle material	4. Strain is zero upto a stress level and then stress remains constant.

Code:

	A	B	C	D
(a)	3	4	2	1
(b)	3	4	1	2
(c)	4	3	2	1
(d)	4	3	1	2

Qu 6 Elastic limit is the point

(a) Up to which stress is proportional to strain

(b) At which elongation takes place without application of additional load

(c) Up to which if the load is removed, original volume and shape are regained

(d) At which the toughness is maximum

Qu 7 Match List-I (Material) with List-II (Properties) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Isotropic	1. Time dependent stress-strain relation
B. Homogeneous	2. No plastic zone
C. Visco-elastic	3. Identical properties in all directions
D. Brittle	4. Similar properties throughout the volume

Codes:

	A	B	C	D
(a)	3	1	2	4
(b)	4	1	2	3
(c)	3	4	1	2
(d)	4	3	2	1

CHAPTER – 2

SHEAR FORCE AND BENDING MOMENT

STRUCTURAL MEMBERS

- A beam is a structural member of sufficient length compared to lateral dimensions.
- Ties, struts, shafts and beams are all one dimensional or line-elements, where the length is much greater than the depth or width, and have different names depending upon the main action they are designed to resist.
- Thus, ties and struts resist uniaxial tension or compression, shafts resist torque and beams resist bending moments (and shear forces). Beams may be concrete, steel or even composite beam, having any type of sections such as angles, channels, I-section, rectangle, square, hat section etc.

SPAN OF BEAM

- The clear horizontal distance between the supports is called the clear span of the beam.
- The horizontal distance between the centres of the end bearings is called the effective span of the beam.
- If the intensity of the bearing reaction is not uniform, the effective span is the horizontal distance between the lines of action of the end reactions.

TYPES OF SUPPORT

The supports can be classified into following categories:

(a) A simple or free support/Roller support/Rocker support

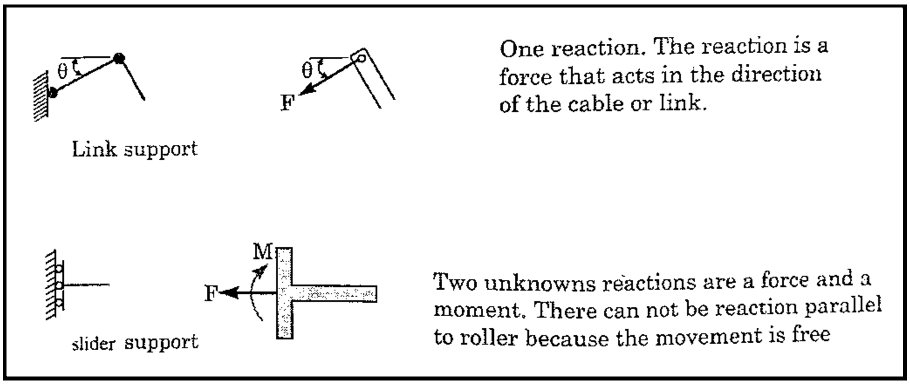
(b) Hinged or pinned support,

(c) A built-in or fixed or encastre support

(d) Slider support

(e) Link support

Other Supports



TYPES OF BEAMS

Depending on the type and number of supports, the beams are divided into two categories:

(i) Statically determinate beam, and

(ii) Statically indeterminate beam.

Statically Determinate Beam (2D)

A beam is said to be statically determinate, when it can be analysed using three equations of static equilibrium i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$, where

ΣF_x = Algebraic sum of horizontal forces

ΣF_y = Algebraic sum of vertical forces

ΣM_z = Algebraic sum of moments of all the forces at a point about z-axis.

Examples of statically determinate beams are as follows:

(i) Cantilevers (ii) Simply supported beams, and (iii) Overhanging beams.

Statically Indeterminate Beam

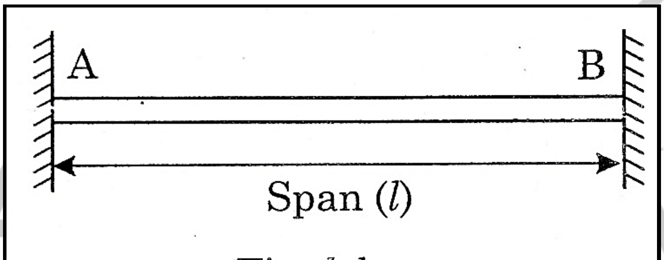
When the number of unknown reactions or stress components exceed the number of static equilibrium equations available, (i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$) the beam is said to be statically indeterminate. This means, that the three equilibrium equations are not adequate to analyse the beam. In this case additional equations of compatibility are required to analyse the beam.

Example are as follows:

(i) Fixed beams (ii) Propped cantilevers, and (iii) Continuous beams.

Fixed Beam

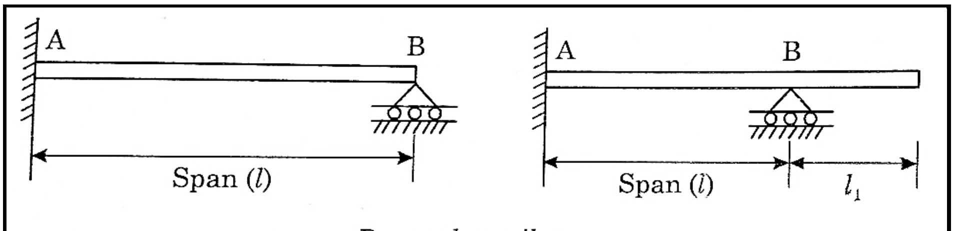
A beam rigidly fixed at its both ends like beam-column rigid/monolithic construction or built-in walls is known as rigidly fixed beam or a built-in beam.



Fixed beam

Propped Cantilever

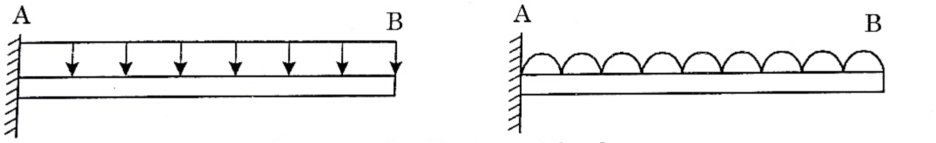
If a cantilever beam is supported by a simple support at the free end or in between, is called propped cantilever. It may or may not be having overhanging portion.



Propped cantilever

Uniformly Distributed Load

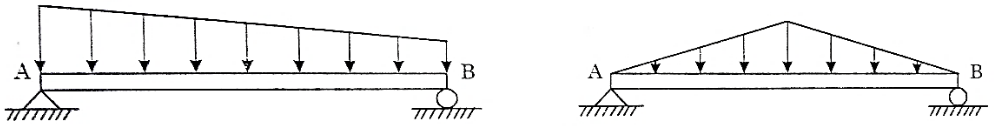
A load which is spread over a beam in such a manner that each unit length is loaded to the same extent, is known as uniformly distributed load (briefly written as u.d.l.).



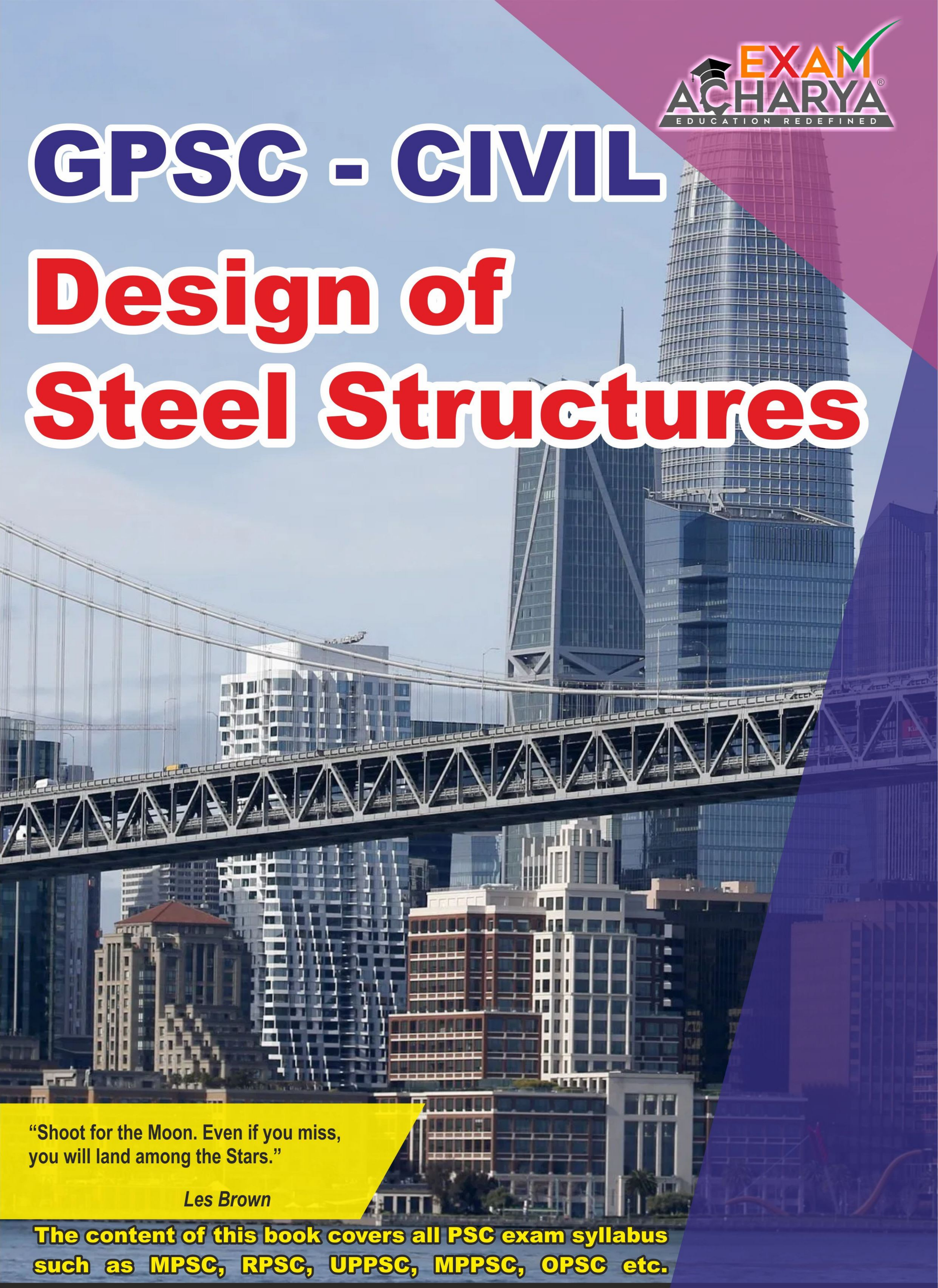
Uniformly distributed load

Uniformly Varying Load

A load which is spread over a beam in such a manner that its extent varies uniformly on each unit length is known as uniformly varying load.



Uniformly varying load



EXAM ACHARYA
EDUCATION REDEFINED

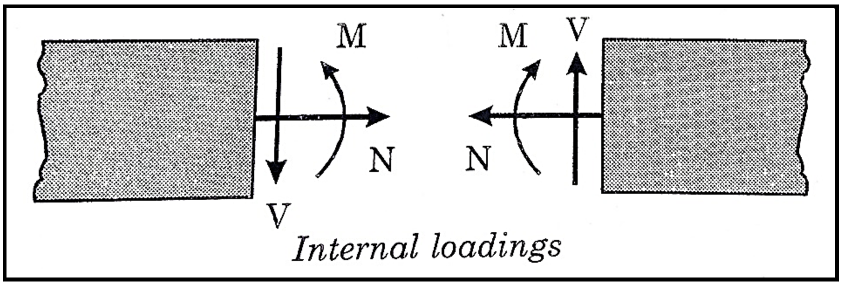
GPSC - CIVIL

Design of Steel Structures

"Shoot for the Moon. Even if you miss, you will land among the Stars."

Les Brown

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.



M = Bending moment at a section

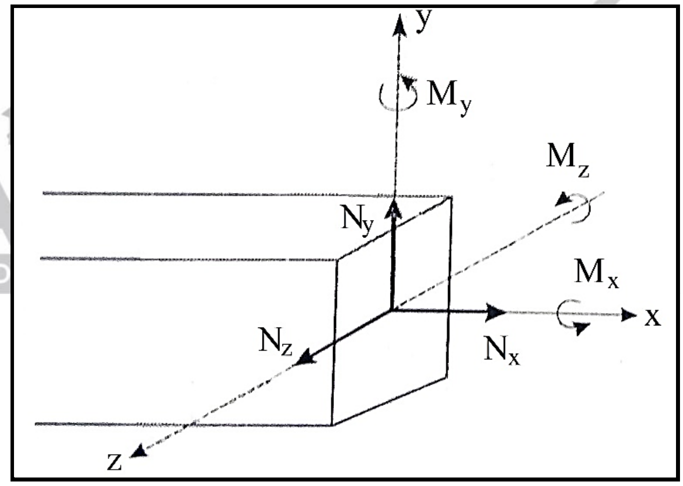
V = Shear force at the section

N = Axial thrust at the section

Note

- For out of plane loading, internal forces are

N_x	N_y	N_z	M_x	M_y	M_z
↓	↓	↓	↓	↓	↓
Axial thrust	Shear	Shear	Torsion	Bending	Bending



If internal loadings are known, stresses at the section can be calculated, (like bending stress $\frac{M.y}{I}$, M=bending moment). Thus, to check the safety of a chosen design section, knowledge of internal loadings at various sections is essential. Hence bending moment and shear force at all sections is found out and their variation is plotted along the length of beam. The diagrams so obtained are called bending moment diagram and shear force diagram.

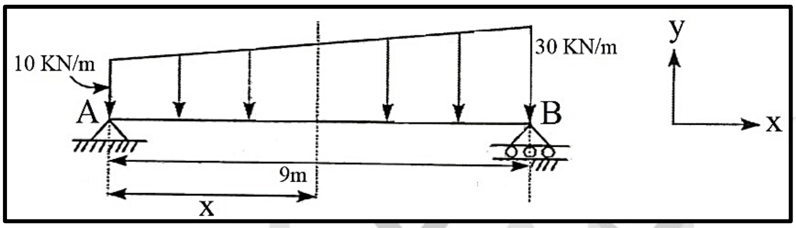
DEFINITION OF AXIAL THRUST

Axial thrust is the force acting along the longitudinal axis of the members. Axial thrust is (+) ve if it tries to elongate the member.

Note
- Shear force is different on either side of concentrated load.
- BM remains the same on either side of concentrated load.

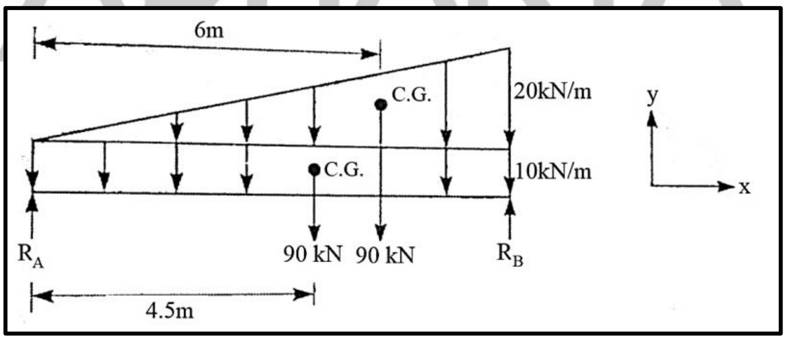
Example

Find BM and SF at a distance ‘x’ from end ‘A’.



Solution

Step 1: Calculate reactions



$\sum F_y = 0, \quad \Rightarrow \quad R_A + R_B = 10 \times 9 + \frac{1}{2} \times 9 \times 20 = 180kN$

$\sum M_A = 0, \quad \Rightarrow \quad R_B \times 9 - 90 \times 6 - 90 \times 4.5 = 0$

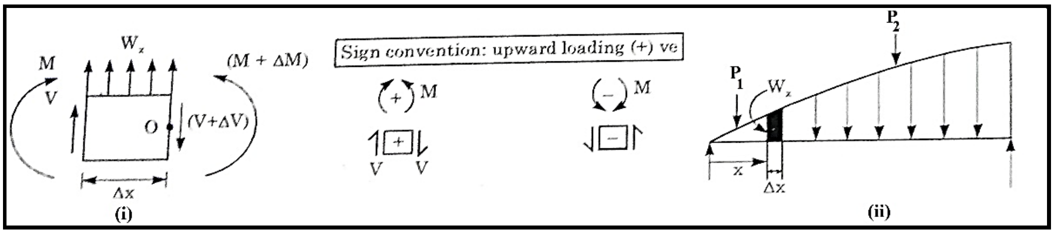
$\Rightarrow \quad R_B = 60 + 45 = 105 \text{ kN}$

$\Rightarrow \quad R_A = 75 \text{ kN}$

M_x = Sagging moment at x

Note
- We will always proceed from left to right i.e., towards increasing value of x, for plotting BMD and SFD.

RELATION BETWEEN BM, SF AND LOADING



For equilibrium $\sum F = 0$,

$$V + W_x \cdot \Delta x - (V + \Delta V) = 0$$

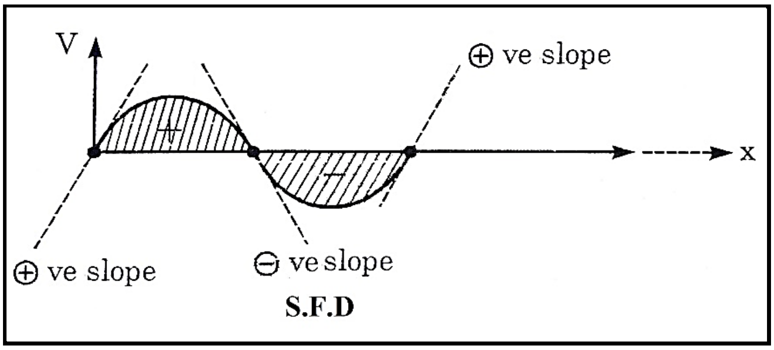
$$\Rightarrow \frac{\Delta V}{\Delta X} = W_x$$

for very small value of x

$$\Rightarrow \frac{dV}{dx} = W_x \quad \text{.......................(i)}$$

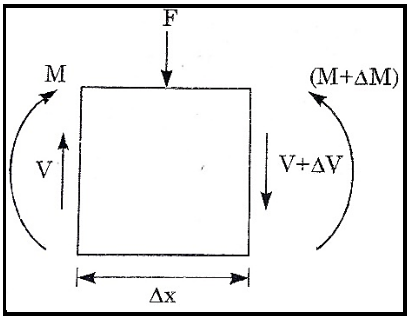
⇒ **Slope of shear force diagram = Intensity of distributed load**

Slope of SFD



If slope of SFD is positive, this implies that load intensity at that point is (+) ve i.e., upwards and if slope of SFD is (-) ve, this implies that load intensity at that point is (-) i.e., downwards.

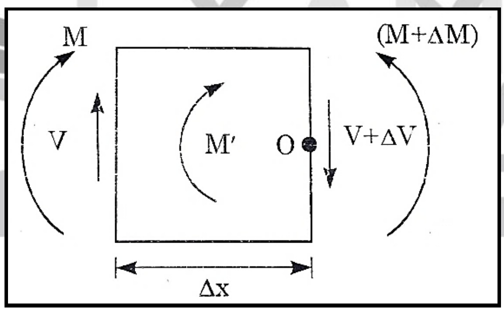
EFFECT OF CONCENTRATED LOAD AND CONCENTRATED MOMENT



$$\sum F = 0, \Rightarrow \quad \Delta V + F = 0$$

$$\Rightarrow \quad \Delta V = -F$$

- When F acts downward at a section, SF drops by amount F at that section
- When F acts upward at a section, S.F jumps up by amount 'F' at that section



$\sum M_0 = 0 \Rightarrow M + V\Delta x + M' - M - \Delta M = 0$

{ M′ = concentrated moment of section}

$$\Rightarrow \qquad \mathbf{\Delta M = M' + V\Delta x}$$

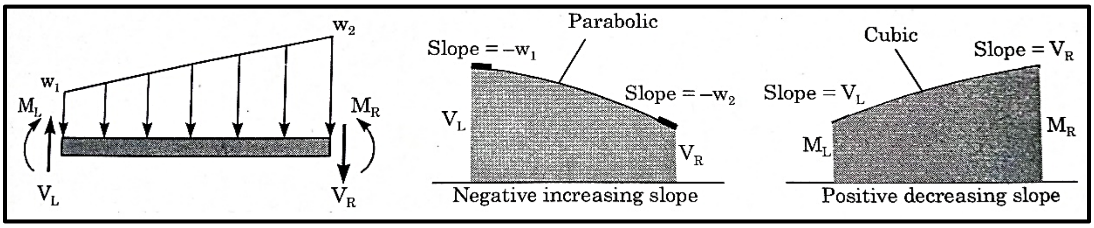
For very small Δx i.e., when $\Delta x \to 0$

$$\mathbf{\Delta M = M'}$$

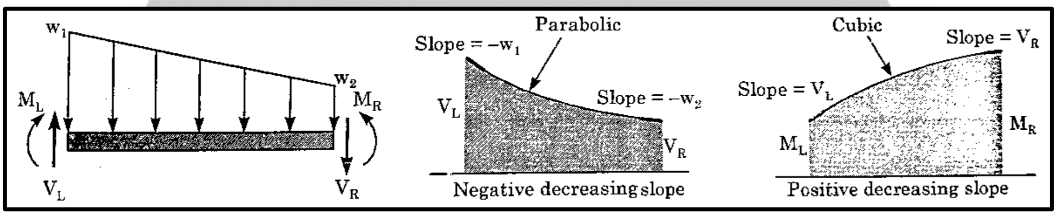
$$\int \mathbf{dM = M' + \int Vdx}$$

$$M_2 - M_1 = \mathbf{M' + \int Vdx}$$

- Loading is (-) ve and constant $\Rightarrow$ SF slope is (-) and constant
- SF is (+) ve decreasing $\Rightarrow$ Bending moment slope is (+) ve decreasing
- If load intensity is udl $\Rightarrow$ SFD is linear $\rightarrow$ BMD is parabolic
- Slope of BMD at any section is equal to SFD ordinate at that section
- Slope of SFD at any section is equal to load intensity at that section
- If load intensity is n-degree curve, SFD will be (n + 1) degree curve and BMD will be (n + 2) degree curve.



- Loading is (-) ve and increasing $\Rightarrow$ SFD slope is (-) ve and increasing
- SF is (+) ve and decreasing $\Rightarrow$ BMD slope is (+) ve and decreasing
- If load intensity is uvl (uniformly varied load) SFD is parabolic and BMD is cubic



SF is (+) ve and decreasing $\Rightarrow$ BMD slope is (+) ve and decreasing

Loading is (-) ve and decreasing $\Rightarrow$ SFD slope is (-) ve and decreasing.

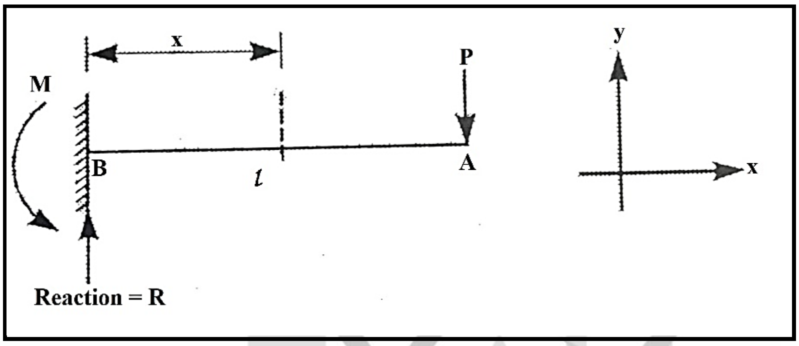
ADDITIONAL POINT

1. When V = 0 i.e., $\frac{dM}{dx} = 0$, B.M. is max/min. Also when shear force changes sign, BM is max at that section.

Example

Draw bending moment diagram and shear force diagram for the following beams.

Solution



Step 1: Calculate reaction

$\sum F_y = 0 \Rightarrow R - P = 0$

$\sum M_B \circlearrowleft = 0 \Rightarrow Pl - M = 0 \Rightarrow M = Pl$ (Hogging)

Step 2 : Analysis of shear force

- At B, loading is upward [point load]

  $\Rightarrow$ SF jumps down by, R = P

- B to A, load intensity = 0 $\Rightarrow \frac{dV}{dx} = w = 0$

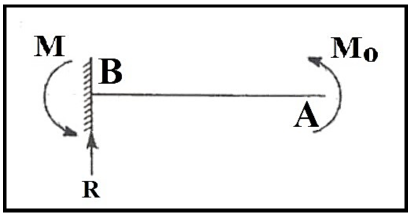
  $\Rightarrow$ Slope of SFD = 0

- At A, there is a downward point load (P)

  $\Rightarrow$ SF jumps down by P

Solution

Step 1: Calculate reaction

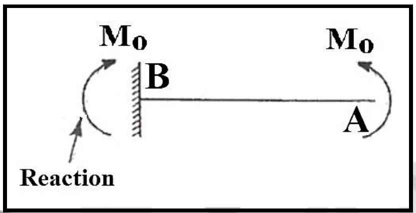


$$\sum F_y = 0 \qquad \Rightarrow R = 0$$

$$\sum M_B \circlearrowleft = 0 \qquad \Rightarrow -M_0 - M = 0$$

$$\Rightarrow \qquad M = -M_0$$

Hence the net reaction is as shown below



Step 2: Analysis of SFD

As there is no loading force anywhere

$\Rightarrow$ SFD = 0

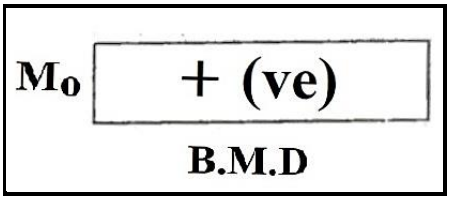
Step 3: Analysis of BMD

At B: M = M_0 [(+) ve means Sagging)]

B to A: $\frac{dM}{dx}$ = V = 0 $\Rightarrow$ BM is constant

At A: Anticlockwise concentrated moment $\Rightarrow$ BMD drops by M_0

BMD is as shown below



Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
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Qu3 The bending moment diagram of the beam shown in the figure-I is

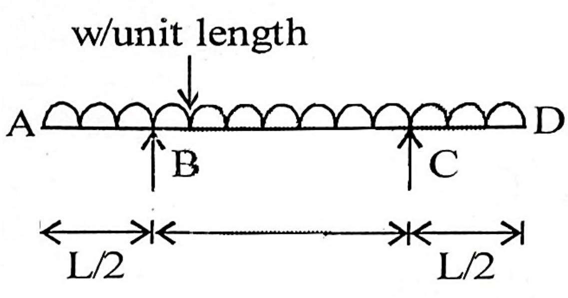
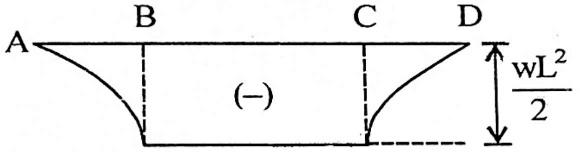
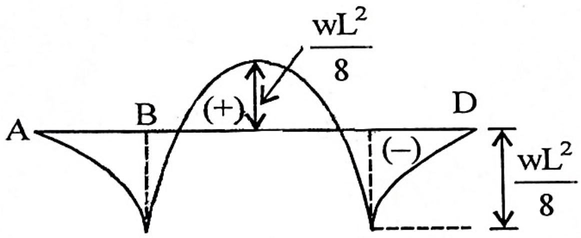
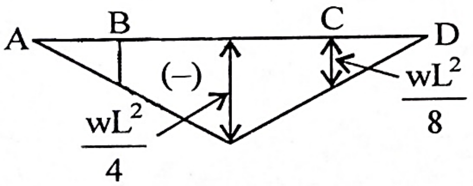
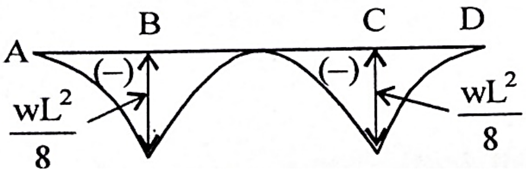


Figure-I

(a)


(b)


(c)


(d)


Qu4 In which one of the following, the point of contraflexure will NOT occur?

(a) A two-span continuous beam of equal spans, simply supported and loaded by UDL over both spans

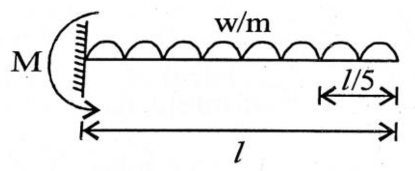
(b) A simply supported beam loaded by UDL

(c) A fixed beam loaded by UDL

(d) A propped cantilever loaded by UDL

TEST YOUR SELF

Qu7 In the given figure the maximum bending moment at the fixed end of the cantilever caused by the UDL is M. The bending moment at a section //5 from the free end is



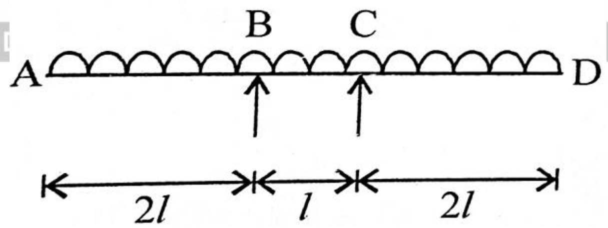
(a) 4% of M

(b) 5% of M

(c) 10% of M

(d) 20% of M

Qu8 ABCD is a beam of length 5*l* which is supported at B and C (having supported length BC = *l*) and having two equal overhangs AB and CD of length 2*l* each. It carries a uniformly distributed load of intensity 'w' per unit length throughout the beam as shown in the given figure.



The points of contra-flexure will occur

(a) At B and C

(b) At the mid-point of BC

(c) Nowhere in the beam

(d) At the mid-points of AB and CD

CHAPTER - 3

DEFLECTION OF BEAM

DEFLECTION DIAGRAMS AND THE ELASTIC CURVE

Deflection of structures can occur from various sources, such as loads, temperature, fabrication errors, or settlement. In design, deflections must be limited in order to prevent cracking of attached brittle materials such as concrete or plaster. Hence deflection calculations are required to estimate whether the deflection at any point is less than the permissible value or not. Moreover, in the analysis of indeterminate structures, deflection at a specified point on a structure is required to write down the compatibility conditions.

Deflection calculation will be done assuming linear elastic material response. [i.e. stress proportional to strain or load is proportional to displacement].

- Deflection of a structure is caused by its internal loadings such as

  Normal force

  Shear force

  Bending moment

  Torsion

- For Beams and frames, major deflection is caused by bending.
- For Trusses, deflection is caused by internal axial forces.

Solution

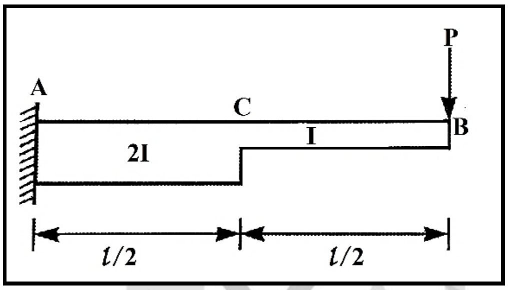
Deflection and slope at B will be calculated from the same approach as above.

$$\theta_C = \frac{Wa^3}{6EI} = \theta_B$$

$$\Delta_B = \frac{Wa^4}{8EI} + \frac{Wa^3}{6EI}(L - a)$$

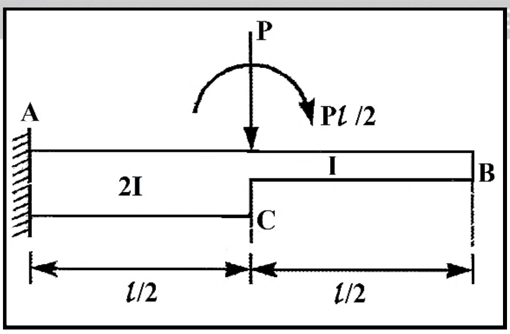
Example

Find Δ_C and θ_C



Solution

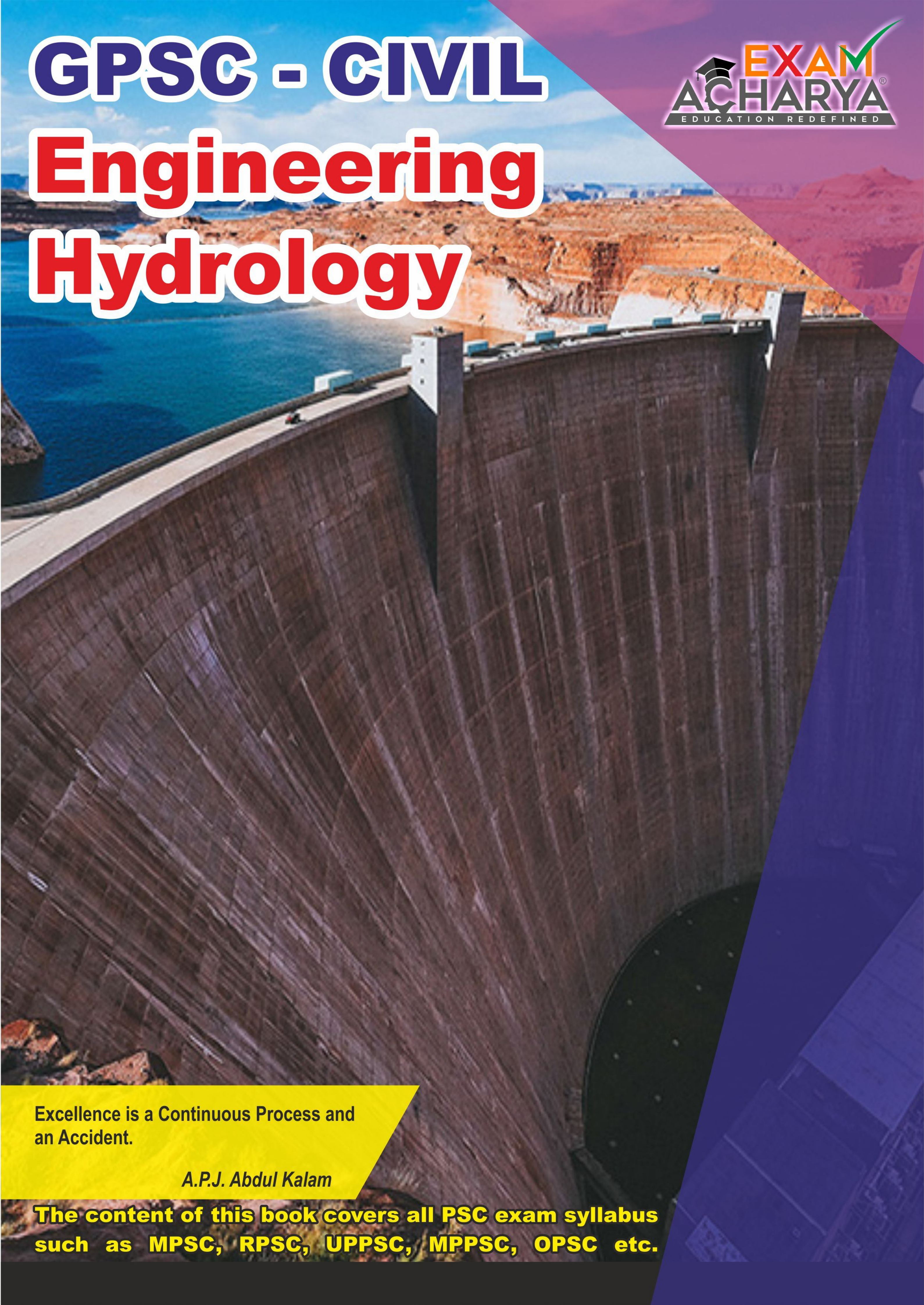
Effect of load at B can be transferred to C as shown in the following figure.



$$\triangle_C = \frac{P\left(\frac{l}{2}\right)^3}{3E(2I)} + \frac{\left(\frac{Pl}{2}\right)\left(\frac{l}{2}\right)^2}{2E(2I)}$$

$$\triangle_C = \frac{Pl^3}{48EI} + \frac{Pl^3}{32EI}$$

$$\triangle_C = \frac{5Pl^3}{96EI}$$



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Engineering Hydrology

Excellence is a Continuous Process and an Accident.

A.P.J. Abdul Kalam

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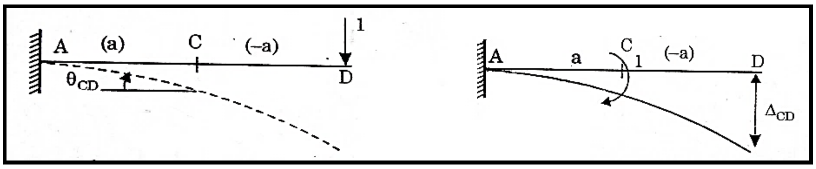
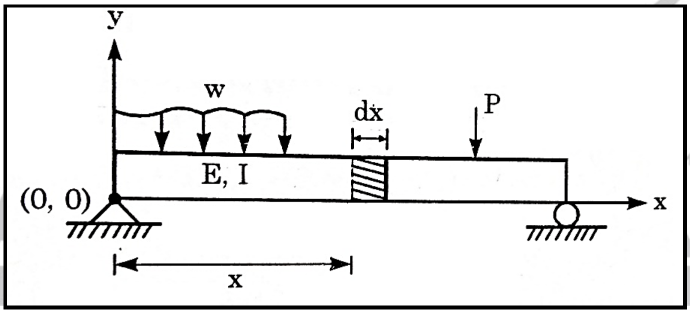


i.e. clockwise rotation at C due to downward unit load at D is equal to downward deflection at point D due to clockwise unit couple at C

$$\theta_{CD} = \frac{a^2}{2EI} + \frac{(L-a)a}{EI} \qquad \Delta_{DC} = \frac{1 \text{ x } a^2}{2EI} + \frac{a(L-a)}{EI}$$

METHODS FOR DETERMINING SLOPE AND DEFLECTION AT A POINT

Double Integration Method



$$\frac{M}{EI} = \frac{\frac{d^2Y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Since slope of elastic curve $\left(\frac{dy}{dx}\right)$is very small, the above equation can be reduced to

$$\mathbf{\frac{d^2Y}{dx^2} = \frac{M}{EI}} \quad \text{..........................(i)}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{M}{EI}$$

$$\mathbf{\frac{d\theta}{dx} = \frac{M}{EI}} \quad \textbf{..............................(ii)}$$

Sign Convention (I)

1. 

Sagging moment is taken as (+) ve

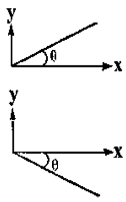
2. 

Hogging moment is taken as (-) ve.

3. (Origin) $\rightarrow$ x (+) ve [x is taken (+) ve when measured from left to right]

4. Upward deflection is taken as (+) ve.

5. Downward deflection is taken as (-) ve.

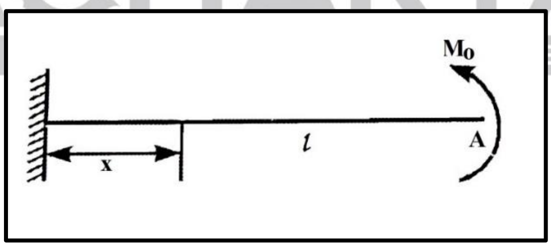
6. 

Slope is taken as (+) ve, if θ is measured counter clockwise from x axis.

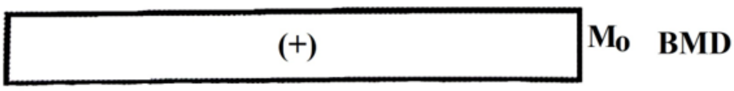
Slope is taken as (-) ve, if θ is measured clockwise from x-axis.

Example

Find slope and deflection at A

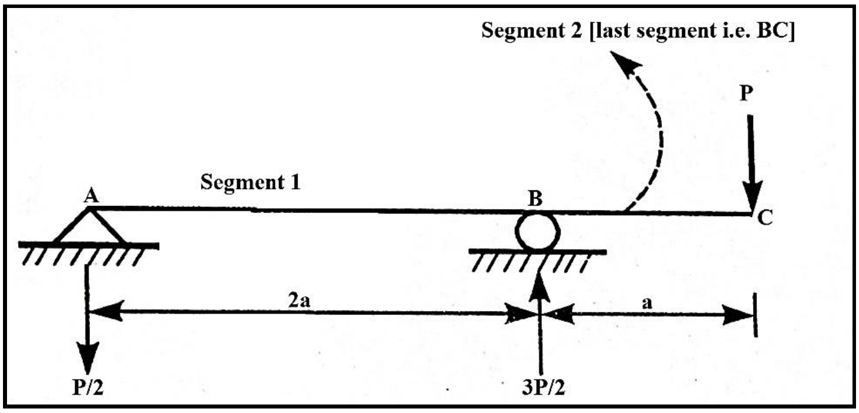


Solution



$$EI\frac{d^2Y}{dx^2} = M_0$$

$$EI\frac{dy}{dx} = M_0x + C_1$$



$$EI\frac{d^2y}{dx^2} = M = \frac{-Px}{2} + \frac{3P}{2}\ (x-2a)\text{---------------(A)}$$

Equation (A) is the BM written for last segment. However, this equation will be valid for the entire span of the beam under the following condition (x - 2a) team is zero if it is (-) ve. i.e., for x < 2a, here x is measured from A i.e., left end support.

$$\Rightarrow EI = \frac{dy}{dx} = \left(-\frac{Px^2}{4} + C_1\right) + \frac{3P}{2}\frac{(x-2a)^2}{2}\text{---------------(B)}$$

$$EI\ y = \left(-\frac{Px^3}{12} + C_1x + C_2\right) + \frac{3P}{2}\frac{(x-2a)^3}{6}\text{------------------(C)}$$

Boundary condition are

At x = 0, y = 0

From (C) $\Rightarrow$ $0 = 0 + 0 + C_2 + 0$

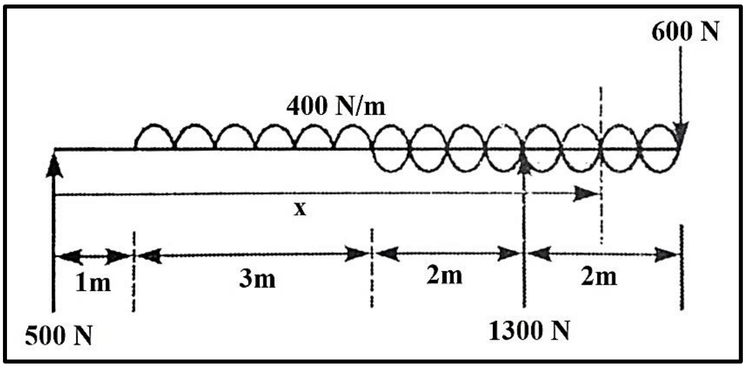
$\Rightarrow$ $\mathbf{C_2 = 0}$

at x = 2a, y = 0

From (C) $\Rightarrow$ $0 = \frac{-P(8a^3)}{12} + C_1(2a) + 0 + 0$

$\Rightarrow$ $C_1 = \frac{+P \times 4a^2}{12} \Rightarrow \mathbf{C_1 = \frac{Pa^2}{3}}$

$$EI\ y = \frac{-Px^3}{12} + \frac{3P}{12}(x-2a)^3 + \frac{Pa^2}{3}x$$



BM equation written for the last segment becomes

$$\mathbf{M = 500\,x - 400\,\frac{\langle x-1\rangle^2}{2} + 400\,\frac{\langle x-4\rangle^2}{2} + 1300\langle x-6\rangle}$$

$$\Rightarrow \ EI\frac{d^2y}{dx^2} = 500\,x - \frac{400\langle x-1\rangle^2}{2} + \frac{400\langle x-4\rangle^2}{2} + 1300\langle x-6\rangle$$

On integration

$$EI\frac{dy}{dx} = \left\{\frac{500x^2}{2} + C_1\right\} - \frac{400\langle x-1\rangle^3}{6} + \frac{400\langle x-4\rangle^3}{6} + \frac{1300\langle x-6\rangle^2}{2}$$

$$EI\,y = \left\{\frac{500x^3}{6} + C_1x + C_2\right\} - \frac{400\langle x-1\rangle^4}{24} + \frac{400\langle x-4\rangle^4}{24} + \frac{1300\langle x-6\rangle^3}{6}$$

Boundary conditions are

At x = 0, y = 0

$\Rightarrow C_2 = 0$ [Because all terms inside < > bracket becomes (-) ve for x = 0 and hence become zero.]

At x = 6m, y = 0

$$\Rightarrow 0 = \frac{500(6)^3}{6} + 6C_1 - \frac{400(6-1)^4}{24} + \frac{400(6-4)^4}{24} + 0$$

$$= 500 \times 36 + 6C_1 - \frac{400 \times 5^4}{24} + \frac{400(2^4)}{24}$$

$$\Rightarrow \quad \boldsymbol{C_1 = -1308.33}$$

Test Series Available..

Total weekly test : 35

Total mid subject test : 16

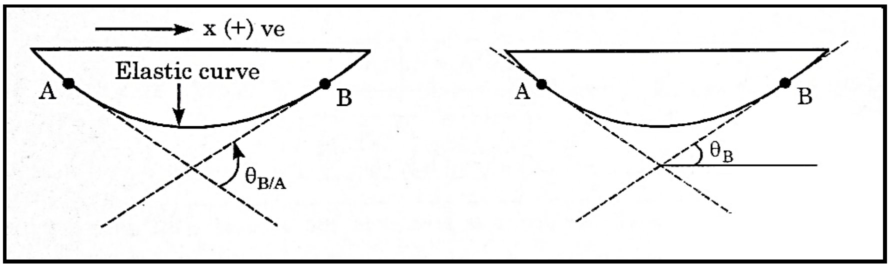
Total full length test : 13

Mock test : 16

Total test : 80

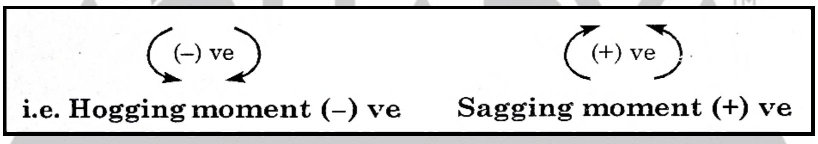
Theorem 1

The change in slope between two points on the elastic curve equals the area of $\frac{M}{EI}$ diagram between these two points.

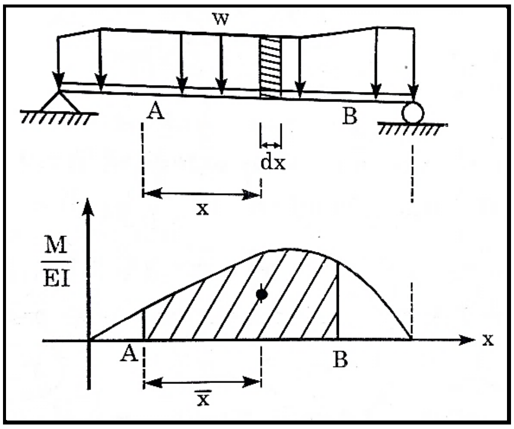


Sign Convention (for slope)

- Angle is measured anticlockwise from tangent at A (i.e., point of smaller x) to tangent at B (i.e., point of larger x) if area of $\frac{M}{EI}$ diagram is (+) ve.
- Angle is measured clockwise from tangent at A to tangent at B if area of $\frac{M}{EI}$ diagram is (-) ve



- Slope at 'B' can be directly known if reference 'A' is chosen such that tangent at 'A' is horizontal i.e. For example, reference point A could be point at fixed support, mid-point of uniformly loaded simply supported beam etc.



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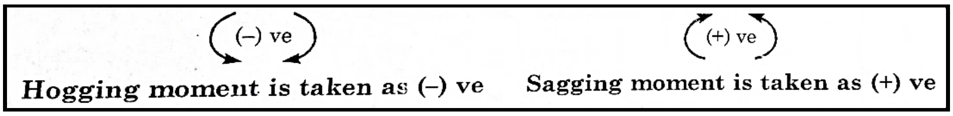
Fluid Mechanics and Hydraulic Machines

"Success Consists of going from Failure without Loss of Enthusiasm."

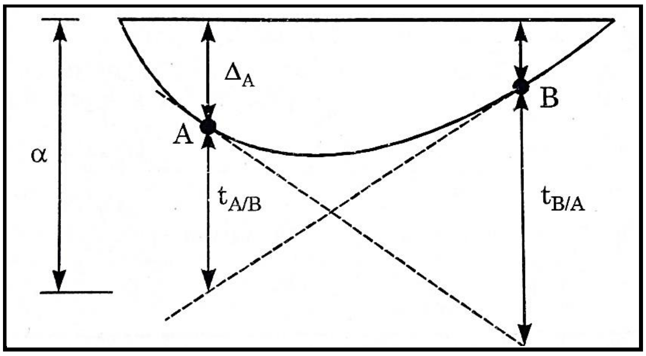
Winston Churchill

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

- Point at 'A' on elastic curve is below tangent extended from 'B' if moment of $\frac{M}{EI}$ diagram between A and B about A is (-) ve.



- $t_{A/B} \neq t_{B/A}$ (generally) [i.e., $t_{AB}/$ is not necessarily equal to $t_{B/A}$]

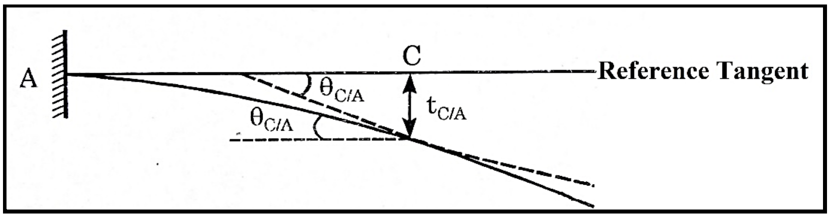


t_{AB} is not necessarily equal to deflection of beam at 'A'. In the above figure if 'α' is known $\triangle_A = \alpha - t_{A/B}$ can be calculated.

$\triangle_A$= deflection of beam at 'A', Numerically.

Application of Moment Area Theorem for Cantilever Beams and Beams with Symmetric Loading

- In cantilever beams, at fixed support, slope of deflected curve is zero. Hence it is taken as reference tangent.



In this case as the reference tangent is horizontal, hence $t_{CA} = \triangle_C$ and slope at C i.e., $\theta_C = \theta_{CA}$

- For symmetrical loading in a simply supported beam, the slope of deflected curve at mid span is zero hence it is taken as reference tangent.

$$= \frac{7250}{\left(2 \times 10^5 \frac{N}{mm^2} \times 4 \times 10^6 mm^4\right) \times 10^{-6}} m$$

$$= 9.06 \times 10^{-3} m = 9.06 \text{ mm}$$

t_{CA} is (+) ve $\Rightarrow$ point C is above tangent from A $\Rightarrow$ upward deflection

$$t_{B/A} = \triangle_B = \frac{250}{EI_{BC}} \times 4 \times 2 = 2.5 \times 10^{-3} m$$

$$= 2.5 \text{ mm}$$

t_{BA} is (+) ve $\Rightarrow$ above tangent from A $\Rightarrow$ upward deflection

Moment Diagram by Parts

- When no. of loadings on a beam is large, it is difficult to calculate the area under resultant $\frac{M}{EI}$ diagram and corresponding e.g., of M/EI diagram.
- We know that resultant BM at any section is the algebraic sum of bending moments at that section caused by each loading separately [either from left or right of that section]. Hence, effect of individual load can be considered, instead of taking effects of all the loads together for drawing BMD. By doing so, we get simpler $\frac{M}{EI}$ diagrams and it is easier to calculate Area and C.G. of the individual moment diagrams. This method of drawing BM diagram due to individual load is called BMD by parts.

Calculation of Redundants Using Moment Area Theorem

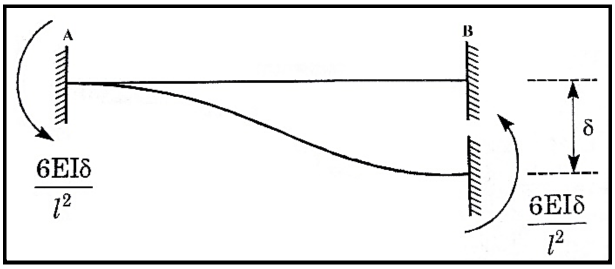
Fixed Beam with Sinking Support

Sinking of support in redundant structures introduces stress in the members of the structure and hence reactions are generated at supports.

Let, due to sinking of fixed support B by an amount $'\delta'$ as shown in figure, moments M_A and M_B and reactions R_A and R_B are generated. Let M_A and M_B be the redundant reactions.

$$\Rightarrow \quad \mathbf{M = \frac{6\,EI\delta}{l^2}}$$

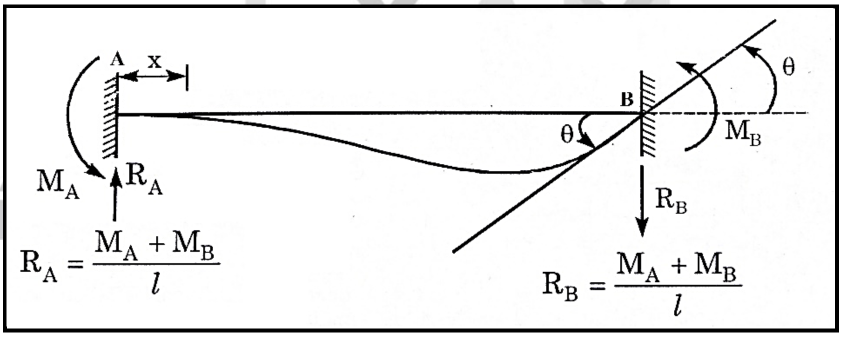
Hence the final result is as shown below.



<u>Fixed Beam with Rotational Slip</u>

Let the fixed support B has rotational slip of anticlockwise nature of magnitude θ.

Rotational slip will also introduce fixed end moment as in the case of settlement of support.



The boundary moment at a distance x from end A is given by M, where

$$M = -M_A + \frac{M_A + M_B}{l}x$$

$$= \left(-M_A + \frac{M_A x}{l}\right) + \frac{M_B x}{l}$$

Hence $\frac{M}{EI}$ diagram can be drawn as

Conjugate Beam Method

The calculation of slope and deflection using area moment theorem required the understanding geometry of deflected shape and was applicable only when the deflected shape was continuous (i.e., no discontinuity in the deflected shape). As a beam with internal hinge has discontinuity in deflected shape, area moment theorem could not be applied to it.

However, in conjugate beam method, we rely only on principle of statics and hence geometry of deflected shape need not be taken into account. Thus, beam with discontinuity in deflected shape (as due to internal hinge or slider) can be easily analysed. On account of these, application of conjugate beam method is simpler.

We know that

$$\left[\frac{dV}{dx} = w \qquad \text{or} \qquad \frac{d^2M}{dx^2} = w\right] \ldots\ldots\ldots\ldots\ldots\ldots (A)$$

$$\left[\frac{d\theta}{dx} = \frac{M}{EI} \qquad \text{or} \qquad \frac{d^2y}{dx^2} = \frac{M}{EI}\right] \ldots\ldots\ldots\ldots\ldots\ldots (B)$$

On integration

$$\begin{bmatrix} V = \int w dx \\ \downarrow \qquad \uparrow \\ \theta = \int \frac{M}{EI} dx \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (C)$$

$$\begin{bmatrix} M = \int [\int w dx] dx \\ \downarrow \qquad \uparrow \\ y = \int \int \left[\frac{M}{EI} dx\right] dx \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (D)$$

The basis of conjugate beam method comes from the similarity of equation (A) and (B) as also the similarity of eq. (C) and (D). Thus, if $\frac{M}{EI}$ diagram is taken as, loading diagram, we get a conjugate beam and then,

Slope at any point in real beam = shear at that point in conjugate beam

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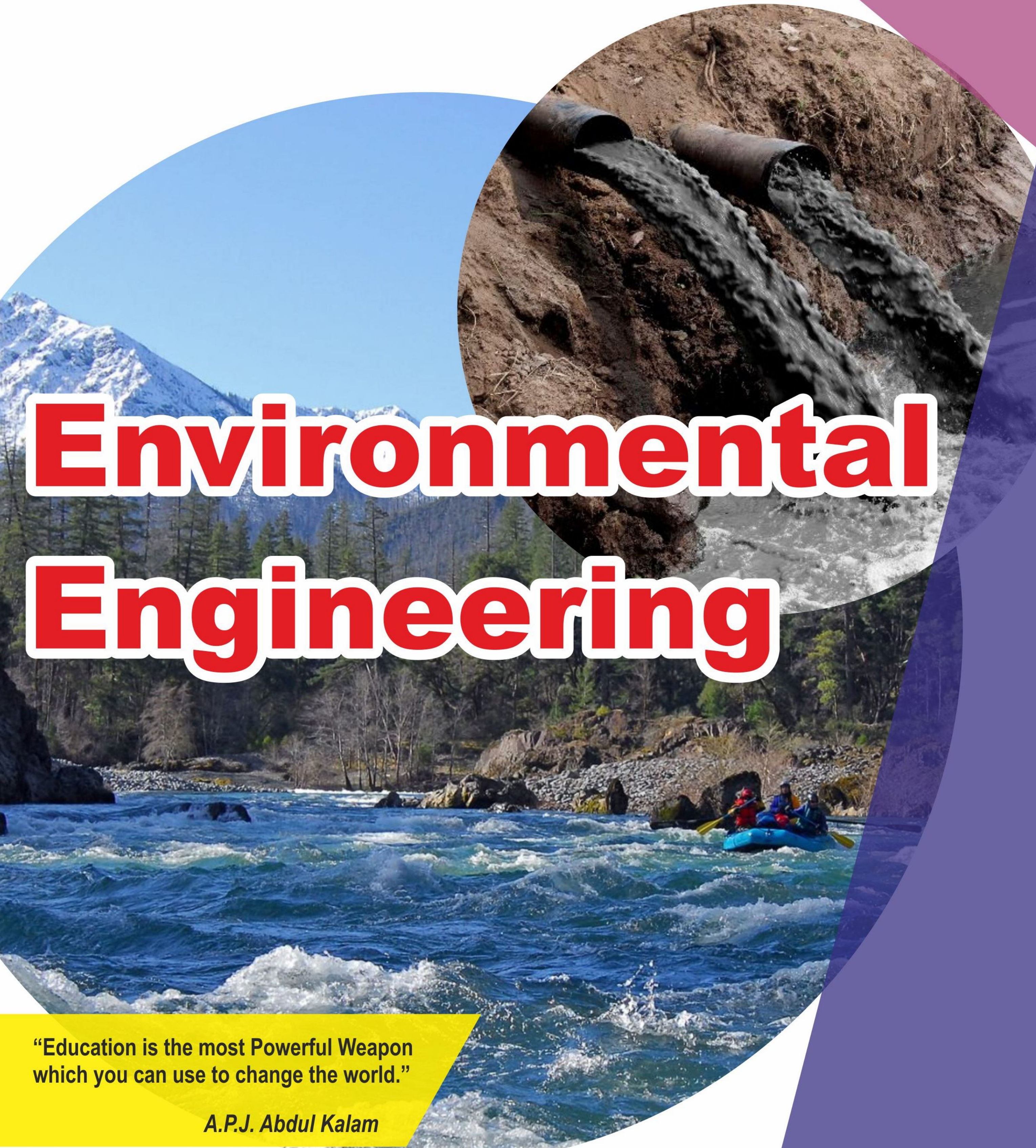


Environmental Engineering

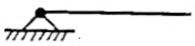
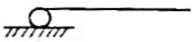
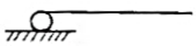
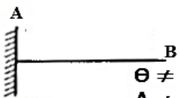
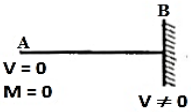
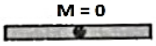
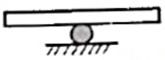
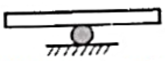
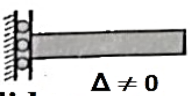
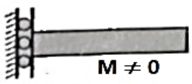
"Education is the most Powerful Weapon which you can use to change the world."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Note that at free end, shear and moment is zero and at fixed end shear and moment will not be zero. Thus, fixed end in beam is replaced by free end in conjugate beam and free end in real beam is replaced by fixed end in conjugate beam.

Hence, the real beam can be converted to a conjugate beam by changing the support conditions as shown below

Real Beam	Conjugate Beam
 **Pin** $\theta \neq 0$, $\Delta = 0$	 **Pin** $V \neq 0$, $M = 0$
 **Roller** $\theta \neq 0$, $\Delta = 0$	 **Roller** $V \neq 0$, $M = 0$
 A: $\theta = 0$, $\Delta = 0$ **Fixed**; B: $\theta \neq 0$, $\Delta \neq 0$ **Free**	 A: $V = 0$, $M = 0$ **Free**; B: $V \neq 0$, $M \neq 0$ **Fixed**
$\theta \neq 0$, $\Delta = 0$  **Internal Pin**	$V \neq 0$, $M = 0$  **Hinge**
$\theta \neq 0$, $\Delta = 0$  **Internal Roller**	$V \neq 0$, $M = 0$  **Hinge**
θ, Δ } exist  **Internal Hinge**	V, M } exist  **Internal Roller**
 **Slider** $\Delta \neq 0$, $\theta = 0$	 **Slider** $M \neq 0$, $V = 0$

Sign Convention

1. If $\frac{M}{EI}$ is (+) ve→ conjugate beam loading is upward
2. If $\frac{M}{EI}$ is (-) ve →conjugate beam loading is downward.

Note
➢ As we have been plotting (+) ve BM above the beam and (-) ve BM below the beam, we can say that conjugate beam loading will always be away from the beam.

3. 

Hogging BM = (-) ve

Sagging BM = (+) ve

4. 

Positive Shear

Negative Shear

5. If shear in conjugate beam is (+) ve ⇒ slope is anticlockwise
6. If moment in conjugate beam is (+) ve ⇒ deflection is upward.
7. Upward deflection is (+) ve and downward deflection is (-) ve.
8. Anticlockwise slope is (+) ve and clockwise slope is (-) ve.

Deflection Using Energy Methods

Principle of work and energy is applied to determine deflection/rotation at a point in a structure. To understand the concept of work and energy, let us discuss certain basic concepts first.

1. <u>Principle of Conservation of Energy</u>

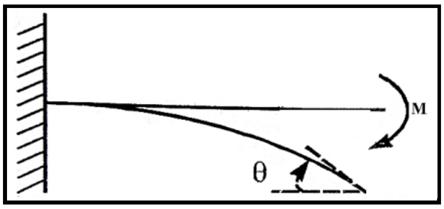
The work done by all the external forces acting on a structure (U_e) is transformed into internal work or strain energy (U_i), which is developed when the structure deforms.

$$\Rightarrow \quad \mathbf{U_e = U_i}$$

⇒ External work done = Internal strain energy stored

$\Rightarrow \mathbf{U_e} = \frac{\mathbf{P\triangle}}{\mathbf{2}}$ Area under load displacement curve i.e. area oab

External work: Due to Moment



⇒ If external moment is gradually applied then external work done is

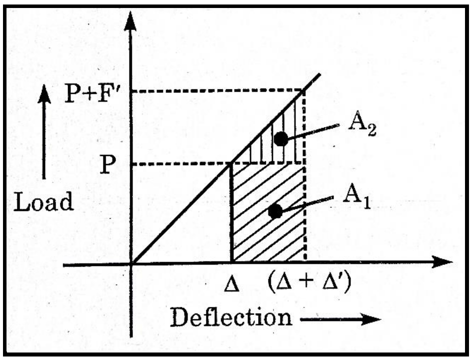
$$U_e = \int_0^\theta M d\theta = \frac{1}{2} M\theta$$

$$\mathbf{U_e} = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{M\theta}$$

If force P is already applied to the bar and another force F′ is now gradually applied so that bar deforms further by amount △'.

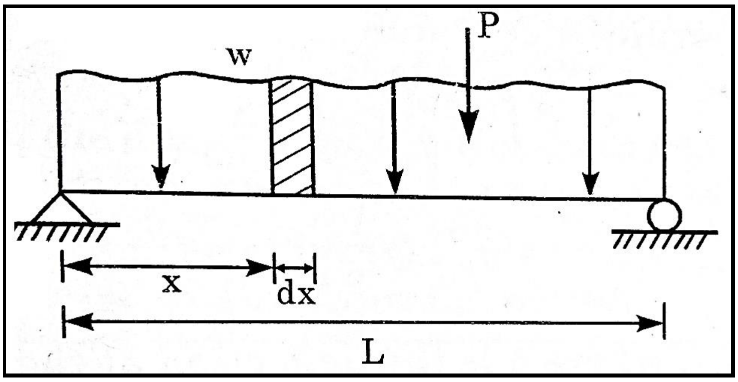
Then work done by P (not F′) during deformation of $\triangle' = P.\triangle'$ = Area A_1, as shown in the following figure.

Work done by F′ = $\frac{1}{2} F' \triangle'$= Area A_2 as shown in the following figure.



Similarly, if moment 'M' is already applied and any other loading deforms the structure by amount θ′, then work done by M = Mθ.'

has no rotation and slope at any point either to the left or right of mid span is summation of rotation of various elements from mid span upto that point.



Gradually applied load P and w creates internal moment M at a distance x from left support. The rotation of differential element dx is dθ. But we know that

$$\mathbf{d\theta = \frac{M}{EI}\,dx}$$

⇒ Internal work done in rotating this differential element from angle 0° to dθ is $\frac{1}{2}M d\theta$. (Since internal moment develops gradually)

⇒ Internal strain energy stored in the differential element dx = dU_i = $\frac{1}{2} M \times d\theta = \frac{1}{2} \times M \times \left(\frac{M}{EI} dx\right) = \frac{M^2 dx}{2EI}$

⇒Total strain energy in the complete beam due to bending $\mathbf{U_i = \int_0^L \frac{M^2 dx}{2EI}}$

Note

- On similar line it can be shown that

$U_i = \int \frac{P^2 dx}{2AE}$ for axial load

$U_i = \int \frac{T^2 dx}{2GJ}$ for torsion

$U_i = K\int \frac{V^2 dx}{2AG}$ for shear

K = Form factor

K depends on the shape of x-section

AE = Axial rigidity

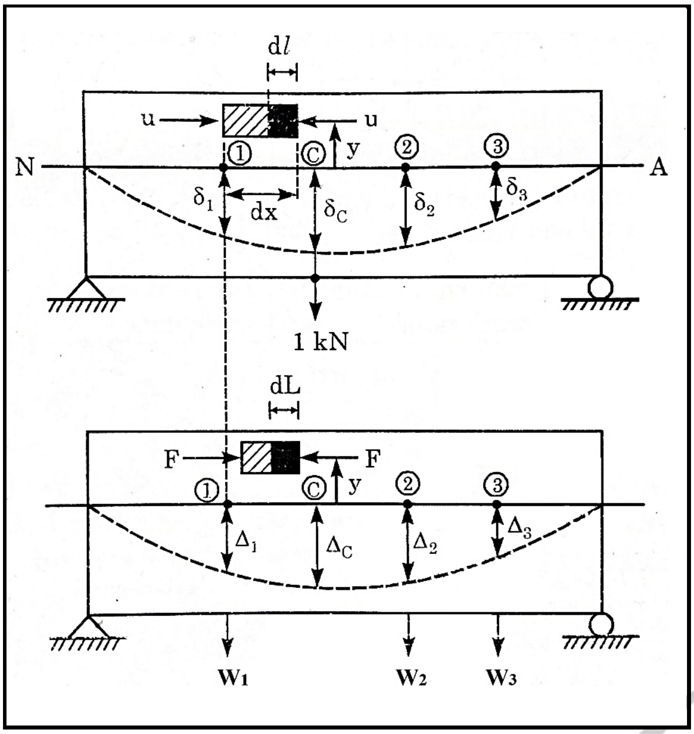
GJ = Torsional rigidity

AG = Shear rigidity

EI = Flexural rigidity

Stiffness = $\frac{\text{rigidity}}{\text{length}}$

Method of Virtual Work (Unit Load Method)



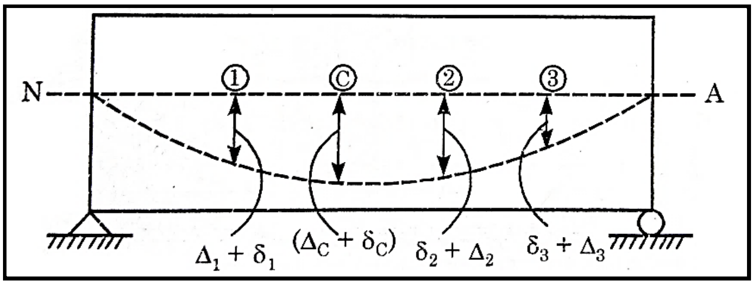
Let $\delta_1, \delta_2, \delta_3$ and δ_C are deflection at point 1, 2, 3 and C due to unit load at C.

u = Internal force developed at any point in beam due to the effect of external unit load. This load 'u' causes a displacement of dl in the differential element of lengths dx as shown in figure.

Let $\Delta_1, \Delta_2, \Delta_3$ and Δ_C are deflection at point 1, 2, 3 and C due to w_1, w_2 and w_3.

F = Internal force developed in the differential element dx in beam due to the effect of external unit load. This load F causes a displacement of dL in the differential element dx.

If unit load is applied 1st and then the external loads (w_1, w_2 and w_3.) are applied, then due to combined action deflections are as shown in fig. below.



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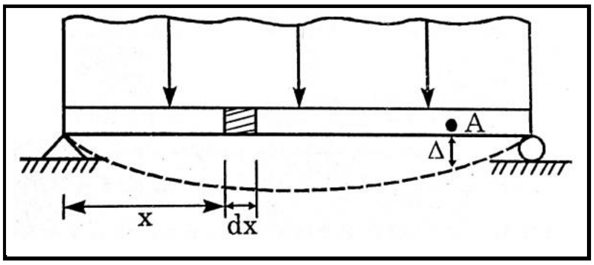
Note
1 = Unit load applied at the point at which deflection is to be found out.
$\triangle_C$ = Deflection/rotation due to external load (i.e., desired deflection).
u = Internal load developed due to unit load.
dL = Deflection of internal element due to external load.

Unit Load Method (Method of Virtual Work): Beams and Frames

Displacement at point ‘A’ is to be determined:



Strain due to bending is the primary cause of deflection in beams and frames. Hence, we take only moment effects here.

For this we follow the steps as described below:

- Apply unit load at 'A' in the direction of desired deflection.
- Due to this unit load, calculate internal virtual moment ‘m’ in beam at section ‘x’.
- When external load is applied, point ‘A’ moves down by amount $\triangle$ and element dx (internal element) deforms or rotates by amount dθ.

  For linear elastic material response

  $$\mathbf{d\theta = \frac{M}{EI} dx}$$

  Where, ‘M’ is the bending moment at ‘x’ due to real loads

  External virtual work done = $1.\triangle$

  Internal virtual work done = $\int_0^L m.d\theta$

Castigliano's Theorem (Method of Least Work)

As per Castigliano's theory

Displacement at a point in a structure is equal to the 1st partial derivative of strain energy in the structure with respect to a force acting at the point and in the direction of desired displacement.

This theorem applies to the structures that have:

1. Constant temperature
2. Unyielding support
3. Linear elastic material response

Hence, $\triangle_i = \frac{\partial U}{\partial P_i}$

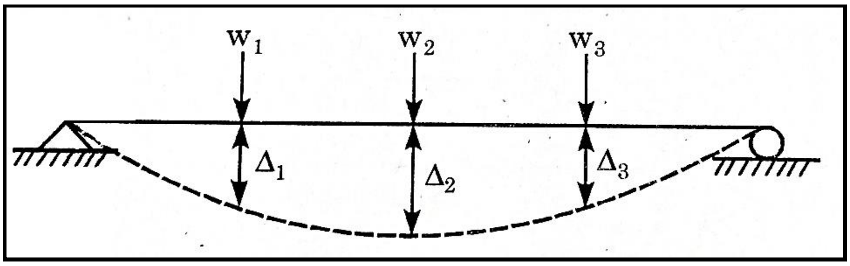
And similarly, $\theta_i = \frac{\partial U}{\partial M_i}$

P_i = Force acting at the point where deflection is to be found out

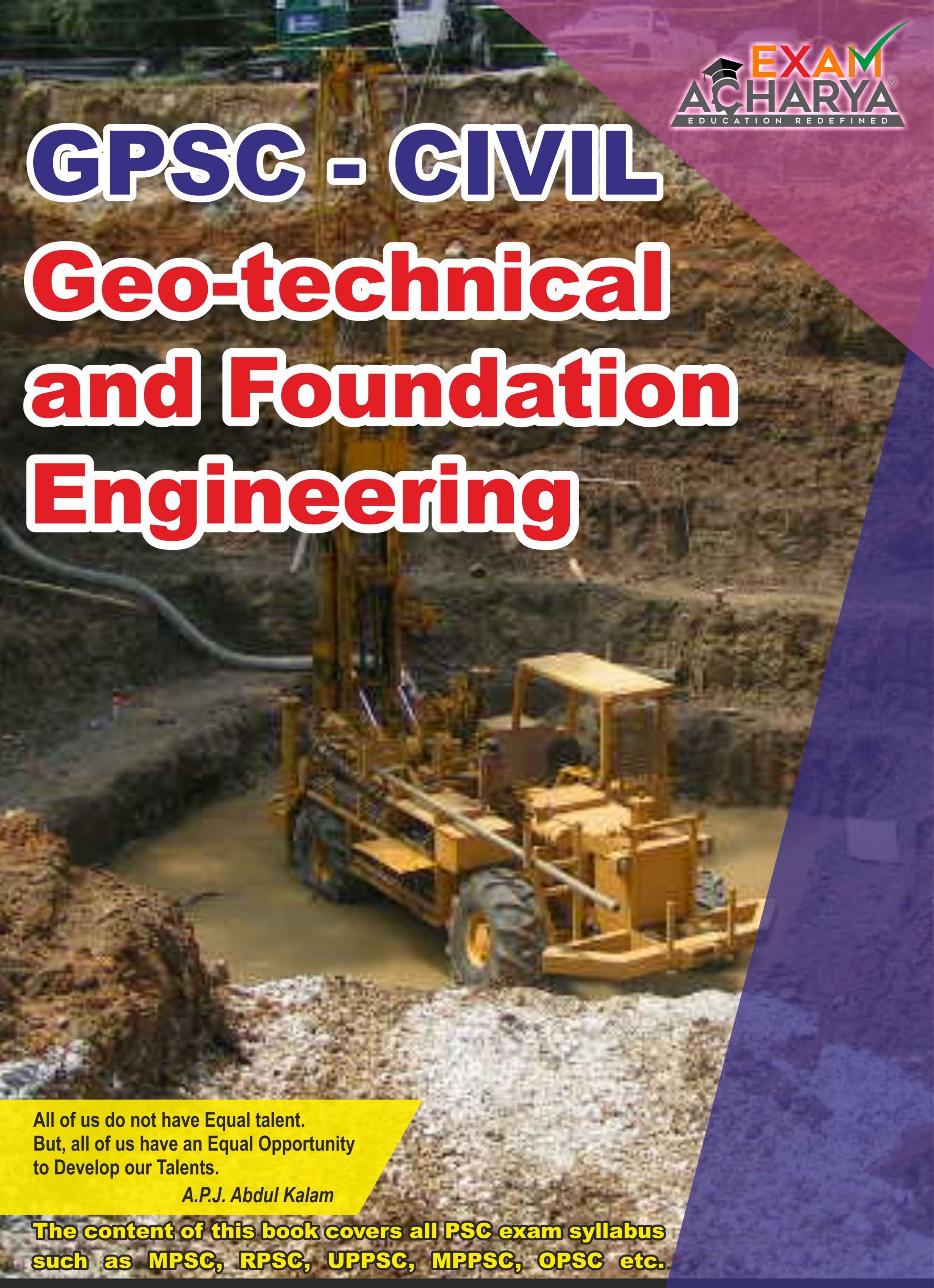
M_i = couple acting at the point where rotation is to be found out

U= Strain energy in the structure due to various forces acting on the structure

Note
- This method is applicable only when forces on the structure are conservative i.e. they do not cause energy losses.
- As against this theorem, method of virtual work also applies to the inelastic behaviour cases. But, theorem applies only to elastic behaviour case.



The external loads, w_1 , w_2 and w_3 acting on a beam produces deflection Δ_1, Δ_2, and Δ_3, (when applied gradually).



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A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

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$$\Rightarrow \quad \frac{1}{2}w_1\partial \triangle_1 + \frac{1}{2}w_2\partial \triangle_2 + \frac{1}{2}w_3\partial \triangle_3 = \frac{dw_{1\triangle_1}}{2} \quad \text{........(iv)}$$

Now from (ii) and (iv)

$$\partial U = \frac{1}{2}dw_1\,\partial \triangle_1 + dw_1.\triangle_1$$

Neglecting smaller term $\frac{1}{2}dw_1\,\partial \triangle_1$ we get

$$\partial U = dw_1.\triangle_1$$

$$\frac{\partial U}{\partial w_1} = \triangle_1 \quad : \textbf{Castigliano's theorem} \qquad \textbf{(A)}$$

This is also called Castigliano's 2nd-theorem.

The 1st theorem is $\frac{\partial U}{\partial \Delta_n} = P_n$

Statement 'A' proves the statement that displacement at a point in a structure is equal to the 1st partial derivatives of strain energy in the structure w.r. to force acting at the point and in the direction of desired displacement.

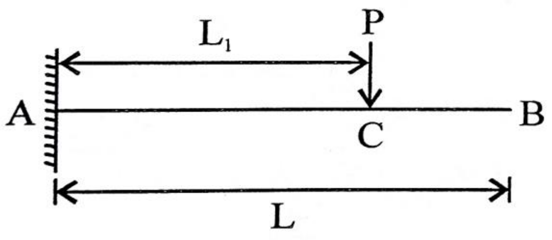
Note

- In this case the deflection $\triangle_1$ was desired only at the point where external load w_1 was acting. If however, deflection is desired at a point where no external load is acting, then we apply an imaginary load(P) at the point of desired deflection and find out strain energy in the system due to combined action of imaginary load (P) and external load and then

$$\triangle = \frac{\partial U}{\partial P}\Big|_{P=0}, \text{ where P is set to zero.}$$

CLEAR YOUR CONCEPT

Qu1 A cantilever carries a load P at C as shown in the given figure.



EI = Constant

The deflection at B is

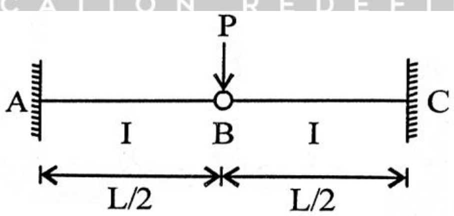
(a) $\frac{PL_1^2}{2EI}(L-L_1)$

(b) $\frac{PL_1^2}{3EI}(L-L_1)$

(c) $\frac{PL_1^2}{2EI}\left(L-\frac{L_1}{3}\right)$

(d)) $\frac{PL_1^2}{3EI}\left(L-\frac{L_1}{3}\right)$

Qu2 What is the deflection at the hinge for the beam shown below?



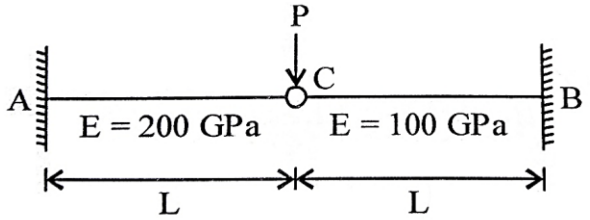
(a) 0

(b) $\frac{PL^3}{3\ EI}$

(c) $\frac{PL^3}{24EI}$

(d)) $\frac{PL^3}{48EI}$

Qu5 What is the bending moment at A for the beam shown below?



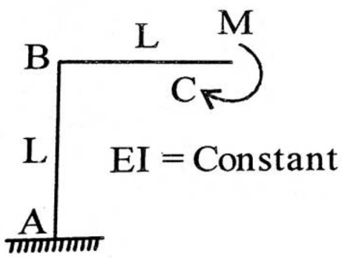
(a) $\frac{PL}{3}$

(b) $\frac{3PL}{2}$

(c) $\frac{PL}{2}$

(d)) $\frac{2\,PL}{3}$

Qu6 What is the horizontal deflection at free end C of the frame shown in the given figure?



a) $\frac{ML^2}{2EI}$

(b) $\frac{ML^2}{EI}$

(c) $\frac{3ML^2}{2EI}$

(d) $\frac{2ML^2}{EI}$

Qu10 A Fixed beam of uniform section is carrying a point load at its mid span. If the moment of inertia of the middle half-length is now reduced to half its previous value, then the fixed end moments will

(a) Increase

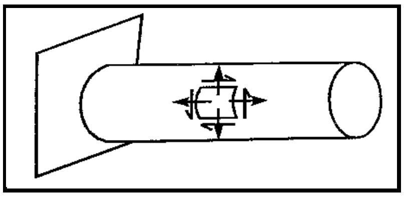
(b) Decrease

(c) Remain constant

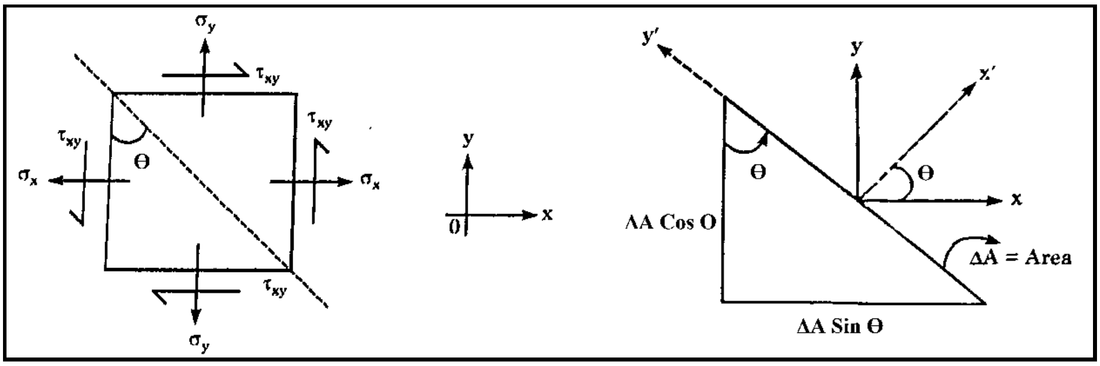
(d) Change their directions

Answer

1-(c), 2-(d), 3-(b), 4-(c), 5-(d), 6-(a), 7-(b), 8-(c), 9-(d), 10-(a)



TRANSFORMATION OF PLANE STRESS



Transformation of stress means we have the stresses $\sigma_x, \tau_{xy}, \sigma_y$ on x and y faces and we have to find out stresses on plane, the normal to which are in x′ and y' directions.

Sign Convention

1. (+) Tension
2. (-) → Compression
3. Shear on (+) ve face in (+) ve direction = (+) ve

   Shear on (-) ve face in (-) ve direction = (+) ve

   Shear on (-) ve face in (+) ve direction = (-) ve

   Shear on (+) ve face in (-) ve direction = (-) ve

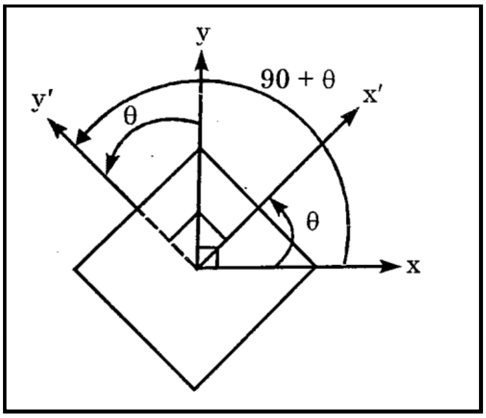
4. Anticlockwise rotation (θ) is taken as (+) ve
5. Clockwise rotation (θ)is taken as (-) ve

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Thus,

$$\boldsymbol{\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta} \text{ ……………..(A)}$$

$$\boldsymbol{\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta} \text{ ………………(B)}$$



As $\sigma_{y'}$ is obtained by replacing θ with θ + 90° in the above equation (A)

and $\cos 2(\theta + 90°) = \cos(180° + 2\theta) = -\cos 2\theta$

$\sin 2(\theta + 90°) = \cos(180° + 2\theta) = -\sin 2\theta$

Hence,

$$\boldsymbol{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta} \text{ ……………..(C)}$$

From (A) and (B)

$$\boldsymbol{\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}} \text{ …………………………..(D)}$$

i.e., In case of plane stress, the sum of normal stresses exerted on a cubic element of material is independent of the orientation of element.

Thus from (B)

$$\Rightarrow \quad \left[\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}\right]\left[\textit{This relation can also be obtained by } \frac{d\sigma_{x'}}{d\theta} = 0\right]$$

From this we get two values of $2\theta_p$ which are separated by 180°because

$$\mathbf{\tan(180° + \alpha) = \tan\alpha}$$

Thus, the two values θ_p are separated by 90° i.e., θ_p and 90 $+\theta_p$. One of these orientations gives max normal stress and other gives min normal stress. Thus, we have two planes on which shear stresses are zero. On one of them normal stress is max and on other it is min.

Major principal stress (σ_{max}) = Max value of normal stress

Minor principal stress (σ_{min}) = Min value of normal stress

Major principal plane → Plane on which σ_{max} acts

Minor principal plane → Plane on which σ_{min} acts

- To find out which θ_p (as discussed above) corresponds to max/min principal stress, we put the value of θ_p in eq. (A)

  $$\text{i.e., in } \sigma'_x = \frac{\sigma_x + \sigma_x}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

  and check whether we get σ_{max} or σ_{min}

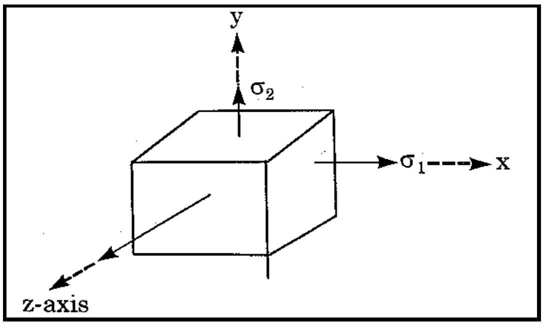
- Point D and E corresponds to max. shear stress

Magnitude of max shear stress = Radius of circle = $\frac{1}{2}$ x difference of principal stresses

$$\boldsymbol{\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}}$$

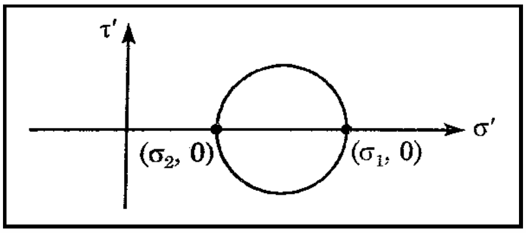
Coordinate of point D is $\left(\frac{\sigma_x + \sigma_y}{2}, \tau_{xy\,max}\right)$

ABSOLUTE MAXIMUM SHEAR STRESS



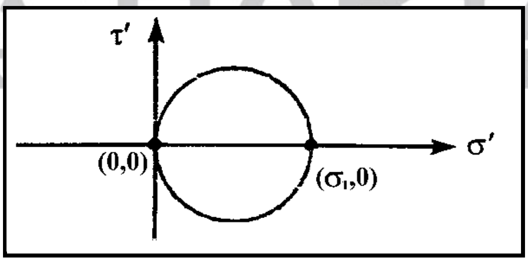
In case of plane stress condition if σ_1 and σ_2 are the principal stresses then

1. For rotation about z-axis the stress condition is shown as



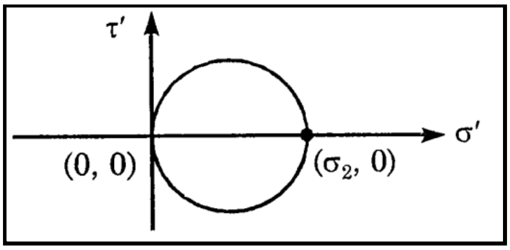
$$\Rightarrow \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

2. For rotation about y-axis the stress condition is shown as



$$\Rightarrow \tau_{max} = \frac{\sigma_1}{2}$$

3. For rotation about x-axis the stress condition is shown as



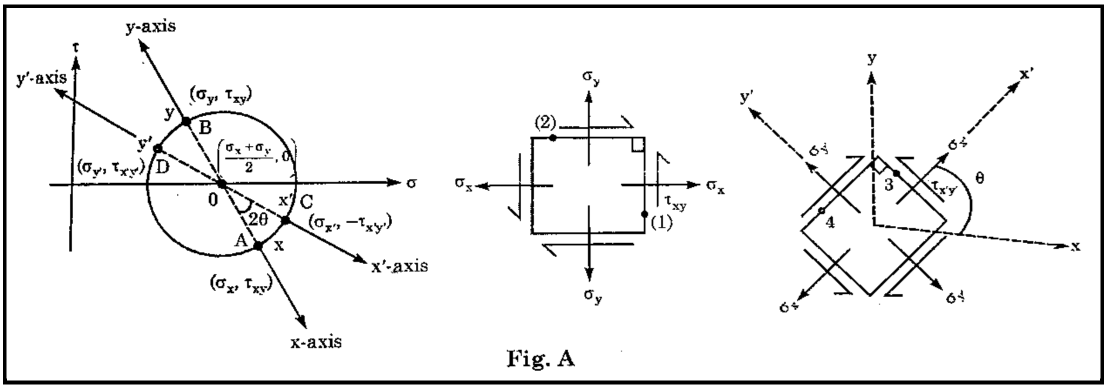


Fig. A

Rules for Applying Mohr's Circle for Transformation of Stress

1. On rectangular $\sigma - \tau$ axis, plot points having coordinates corresponding to face 1 and face 2 adopting the sign convention of Mohr circle. In this case face 1 has co-ordinates $(\sigma_x \ , -\tau_{xy})$ and face 2 has co-ordinate $(\sigma_y \ , \ \tau_{xy})$.

2. Join the points just plotted by straight line. This line is the diameter of a circle whose centre is on the σ-axis.

   [Note that the line joining the two point will be a diameter only when the two points correspond to two faces which are $\perp^r$ to each other].

   If the two planes, the stresses of which have been plotted, are not $\perp^r$ to each other, then to get the centre of circle, a perpendicular bisector is drawn on the line joining the two point. Intersection of this perpendicular bisector with the σ-axis is the centre of the circle.

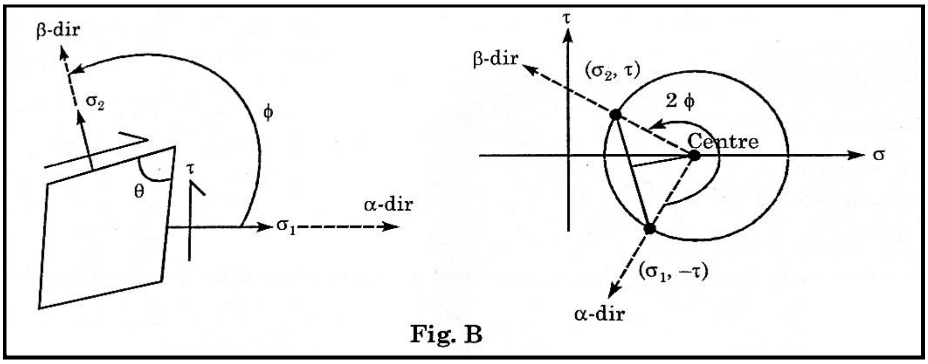


Fig. B

3. As different planes are passed through the selected points in a stressed body, the normal and shearing stress components on these planes are represented by the

Note
- Normal and shear stress are independent. One is not affected by other.

By simplifying equations (I), we get

$$\left.\begin{aligned}\sigma_x &= \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)\\ \sigma_y &= \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)\\ \tau_{xy} &= G.\gamma_{xy}\end{aligned}\right\} \quad (J)$$

Special Cases

(i) **Bi-axial stress condition** (i.e., when only σ_x and σ_y are acting and $\tau_{xy} = 0$).

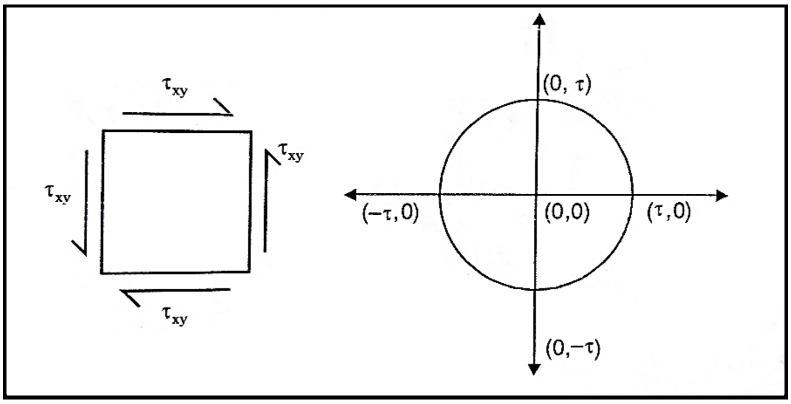
Same equation as derived above in (A) and (B) are applicable because effects of normal and shear stresses are independent of each other.

(ii) **Uniaxial stress** (i.e., when only σ_x is acting and $\sigma_y = 0$ and $\tau_{xy} = 0$)

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu\sigma_x}{E}$$

(iii) **Pure shear case** (i.e., when only τ_{xy} is acting and $\sigma_x = \sigma_y = 0$)



Strain energy due to shear stress, $U_2 = \frac{\tau_{xy} \cdot \gamma_{xy}}{2}$ (per unit volume)

- Total strain energy per unit volume

$U = U_1 + U_2$

$$\mathbf{U = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}\right)}$$

- If σ_1 and σ_2 are principal stresses then strain energy per unit volume is

$$U = \frac{1}{2}[\sigma_1(\varepsilon_1) + \sigma_1(\varepsilon_2)]$$

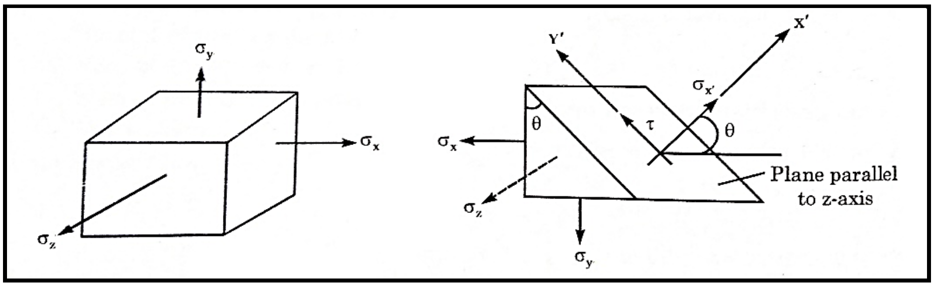
$$= \frac{1}{2}\left[\sigma_1\left(\frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}\right) + \sigma_2\left(\frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}\right)\right]$$

$$\mathbf{U = \frac{1}{2E}\left(\sigma_1^2 + \sigma_1^2 - 2\mu\sigma_1\sigma_2\right)} \text{ (Strain energy per unit volume)}$$

Triaxial Stresses

- An element subjected to σ_x, σ_y and σ_z only [Shear =zero on x,y,z faces] is said to be in triaxial stress.

- σ_x , σ_y and σ_y are principal stresses



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

- In triaxial stress condition also if transformation of stress is done to find out stress on planes obtained by rotation about z-axis the transformation eq. will be same as that under plane stress condition (i.e., eq. A).

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$\Rightarrow$ U=$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\mu\sigma_x\sigma_y - 2\mu\sigma_y\sigma_z - 2\mu\sigma_z\sigma_x$

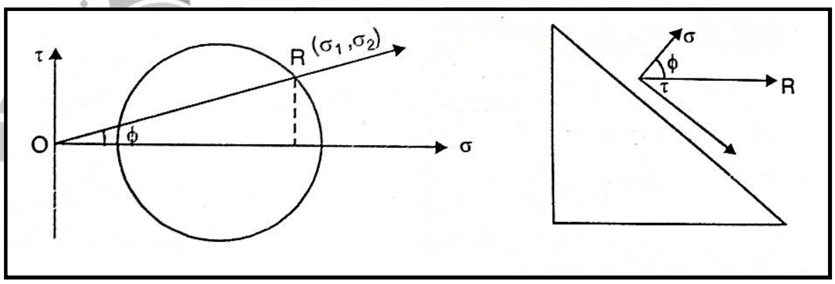
$$\mathbf{U = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\mu(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)}$$

(under triaxial stress condition)

Spherical Stress

[It is a special case of triaxial stress when $\sigma_x = \sigma_y = \sigma_z = \sigma_0$]

- Under this condition any plane cut through the element will be subjected to same normal stress = σ_0.
- Shear stress on all planes = 0
- Every plane is a principal plane.
- Mohr circle reduces to a point.
- Normal strain = $\boldsymbol{\varepsilon_0 = \frac{\sigma_0}{E}(1 - 2\mu)}$



The inclination of the line joining any point P on the Mohr's circle and the origin O will x-axis equal the angle between the resultant stress and the normal of the plane for which P stands.

PLAIN STRAIN

If the only deformations are those in x-y plane then three strain components may exist.

ε_x = Normal strain in x-direction.

ε_y = Normal strain in y-direction.

γ_{xy} = Shear strain associated with x-y plane.

- An element of material subjected to these strain (and only these strains) is said to be in a state of plane strain.
- For plain strain other three components of strain ε_z , γ_{xz} , γ_{yz} are zero.

Comparison of Plane Stress and Plane Strain

	Plane Stress	Plane Strain
Stress	$\sigma_z = 0, \tau_{xz} = 0, \tau_{yz} = 0$ σ_x, σ_y and τ_{xy} may be non-zero	$\tau_{xz} = 0, \tau_{yz} = 0$ $\sigma_x, \sigma_y, \sigma_z$ and τ_{xy} may be non-zero
Strain	$\gamma_{xz} = 0, \gamma_{yz} = 0$ $\varepsilon_x, \varepsilon_y, \varepsilon_z$ and γ_{xy} may be non-zero	$\varepsilon_z = 0, \gamma_{xz} = 0, \gamma_{yz} = 0$ $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ may be non-zero

Note

$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$ $\qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$

$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_z}{E} - \frac{\mu\sigma_x}{E}$ $\qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$

$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$ $\qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$

- If $\sigma_z = 0$, that does not mean $\varepsilon_z = 0$ [except, in case when $\mu = 0$ i.e., ideal material or when $\sigma_x = -\sigma_y$]

  Similarly,
- If $\varepsilon_z = 0$, that does not mean $\sigma_z = 0$ [except in case when $\mu = 0$ i.e. ideal material or when, $\sigma_x = -\sigma_y$]
- Thus, note that plain stress and plain strain components are not same.
- In term of stress, $\sigma_z = 0$ in plane stress but σ_z may not be zero in plane strain.

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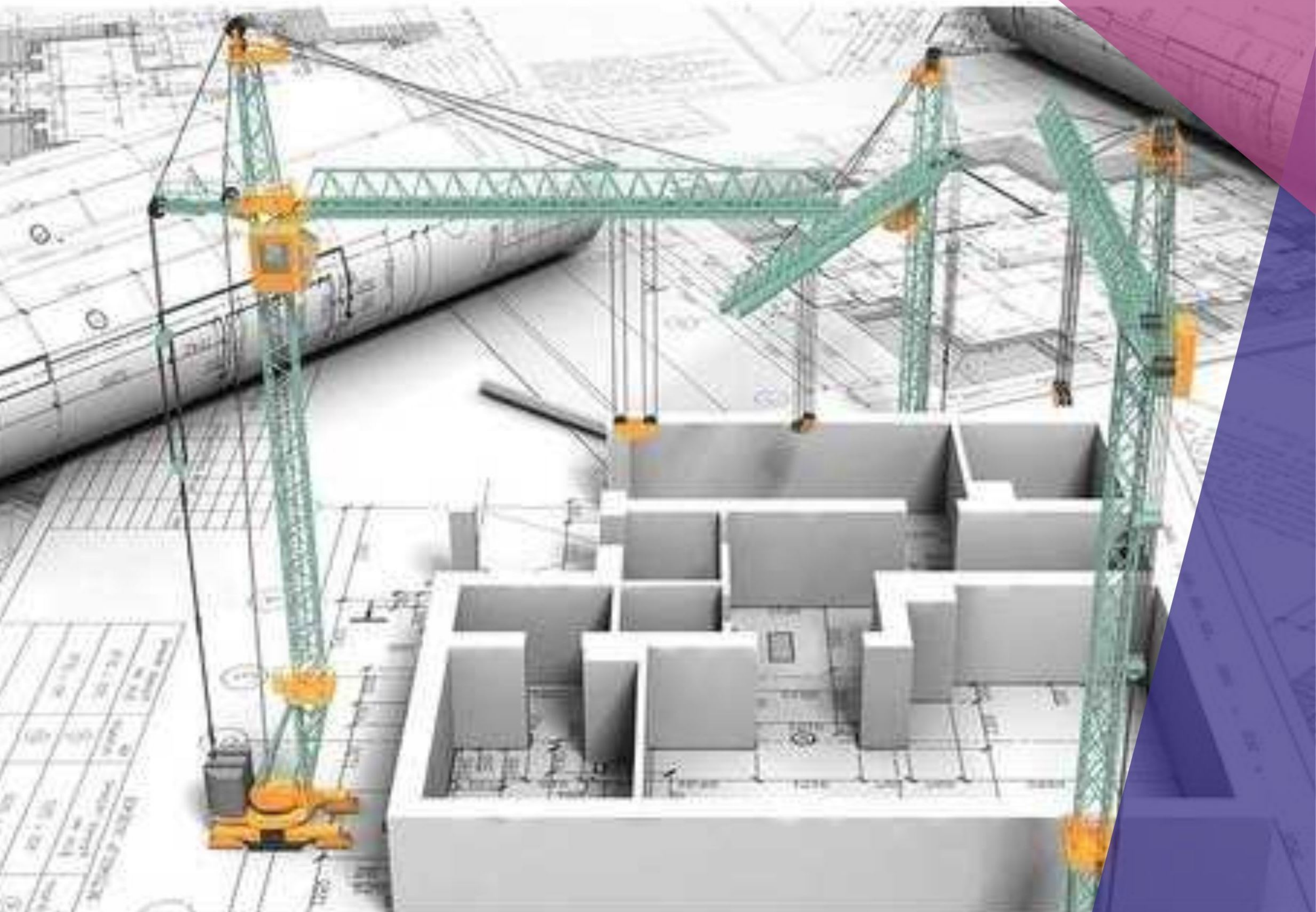


Structural Analysis

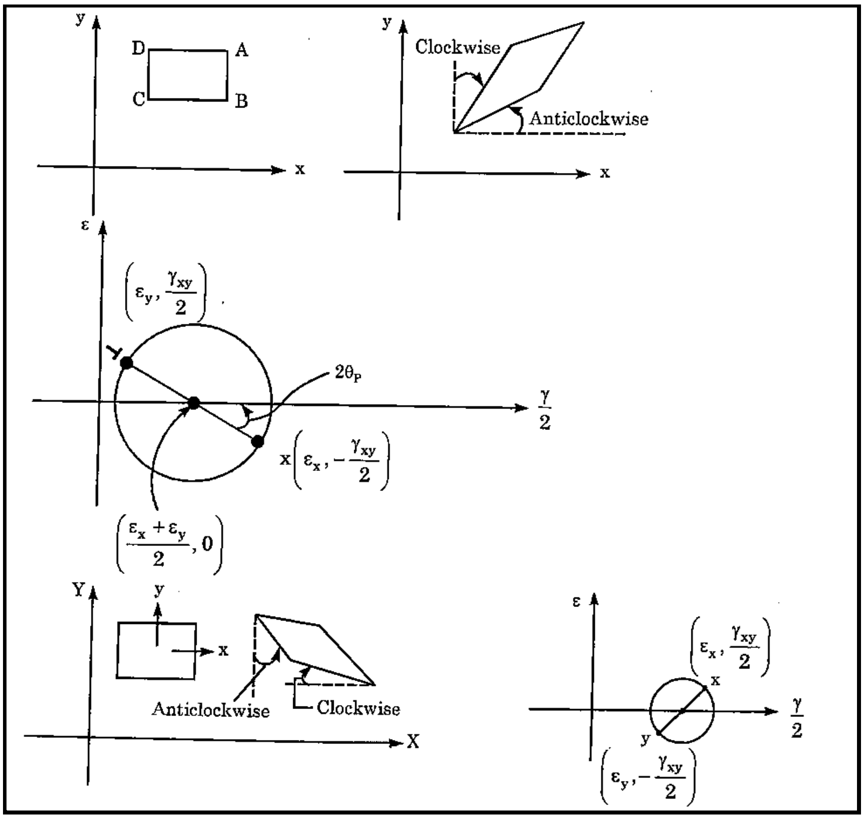
"All of us do not have Equal Talent.
But, all of us have an Equal Opportunity
to Develop our Talents."
A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Shear Strain

- lf shear deformation causes a given side to rotate clockwise, corresponding point on Mohr's circle for plane strain is plotted above horizontal line i.e. (+)ve.

- lf shear deformation causes a given side to rotate clockwise, corresponding point on Mohr's circle for plane strain is plotted below horizontal line i.e. (-)ve.



Note

➢ Strain associated with BC or AD line is taken as ε_x and that associated with AB line is taken as ε_y.

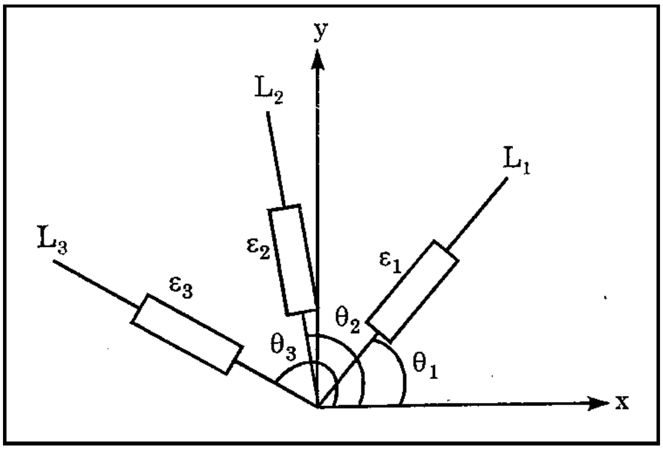
Radius of circle is = $\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

Principal strains are

$$\boldsymbol{\varepsilon_{max/min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\frac{(\varepsilon_x - \varepsilon_y)^2}{2} + \left(\frac{\tau_{xy}}{2}\right)^2}}$$

$$\boldsymbol{\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}}$$

- The transformation equation requires $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ to find out state of strain in various direction. To find out these, three normal strains $\varepsilon_1, \varepsilon_2$ and ε_3 are measured using strain rosette.



Thus,

$$\varepsilon_1 = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1$$

$$\varepsilon_2 = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta_2 + \frac{\gamma_{xy}}{2}\sin 2\theta_2$$

$$\varepsilon_3 = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta_3 + \frac{\gamma_{xy}}{2}\sin 2\theta_3$$

Or

$$\boldsymbol{\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1\cos\theta_1}$$

$$\boldsymbol{\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2\cos\theta_2}$$

$$\boldsymbol{\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3\cos\theta_3}$$

By solving these three equations simultaneously, ε_x , ε_y and γ_{xy} can be calculated.

- The theories of failures proposed in this chapter will be applicable to static loadings only.

The various theories of failure are:

(i) Maximum Principal Stress Theory

(Rankine Theory, Lame's theory or Max Stress theory)

For no failure, maximum principal stress should be less than yield stress under uniaxial loading

$$\text{i.e. } \boldsymbol{\sigma_{max}} \leq \mathbf{f_y}$$

- For design purpose $\boldsymbol{\sigma} \leq \frac{\mathbf{f_y}}{\mathbf{F.o.s.}}$
- This theory is applicable for Brittle material because brittle material fails under tension leading to fracture.
- Not suitable for ductile material in which strength is limited by shear.
- Not suitable for pure shear case because under pure shear, max principal stress = τ (shear stress) hence as per principal stress theory

$$\tau = f_y$$

but in ductile material under pure shear, strength should not exceed $\frac{f_y}{\sqrt{3}}$

(ii) Maximum Principal Strain Theory (St. Venant Theory)

- For no failure, maximum principal strain should be less then or equal to the yield strain in uniaxial loading.

$$\text{i.e.} \qquad \varepsilon_1 \leq \frac{f_y}{E}$$

$$\frac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E} \leq \frac{f_y}{E}$$

where σ_1, σ_2 and σ_3 are principal stresses.

- This method gives the most conservative design out of various other theories of failure.

- Not suitable for hydrostatic loading because under hydrostatic loading when $\sigma_{max} = \sigma_{min} = P$

  $\tau_{max} = 0.$

(iv) Maximum Strain Energy Theory (Beltrami-Haigh Theory)

- Total strain energy per unit volume absorbed at a point should be less than or equal to total strain energy per unit volume under uniaxial loading, when the material is subjected to stress upto elastic limit.

- Total strain energy per unit volume is given by

  $U = \frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_2\varepsilon_2 + \frac{1}{2}\sigma_3\varepsilon_3$ ($\sigma_1, \sigma_2, \sigma_3$ are principal stresses and ε_1, ε_2, and ε_3 are principal strains)

  $$= \frac{1}{2}\sigma_1\frac{[\sigma_1 - \mu(\sigma_2 + \sigma_3)]}{E} + \frac{1}{2}\sigma_2\frac{[\sigma_2 - \mu(\sigma_3 + \sigma_1)]}{E} + \frac{1}{2}\sigma_3\frac{[\sigma_3 - \mu(\sigma_1 + \sigma_2)]}{E}$$

  $$= \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

- Max strain energy per unit volume under uniaxial loading is $\frac{f_y^2}{2E}$

  $$\Rightarrow \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1] \le \frac{f_y^2}{2E}$$

  $$\boldsymbol{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \le f_y^2}$$

For design purpose,

$$\boldsymbol{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1] \le \left(\frac{f_y}{F.o.s.}\right)^2}$$

For 2D:

$$\boldsymbol{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le f_y^2}$$

$\Rightarrow \; 3\tau^2 \le f_y^2$

$$\boldsymbol{\tau \le \frac{f_y}{\sqrt{3}}}$$

Hence this theory is in perfect agreement with the case of pure shear.

Note

- We know that normal stress causes change in volume, but shear stress causes no change in volume, it only causes distortion. Hence out of total strain energy, if strain energy due to volumetric strain is subtracted, we get strain energy due to distortion.
- Volumetric strain energy = $\frac{1}{2}$ x Volumetric stress x Volumetric strain

$= \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\left(\frac{(\sigma_1+\sigma_2+\sigma_3)(1-2\mu)}{E}\right)$

$= \frac{(\sigma_1+\sigma_2+\sigma_3)^2(1-2\mu)}{6E}$

$\Rightarrow$ Volumetric strain energy $= \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2+2\sigma_1\sigma_2+2\sigma_2\sigma_3+2\sigma_3\sigma_1(1-2\mu)}{6E}$

Distortion Energy = Total strain energy – Volumetric strain energy

$= \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right] - \frac{1-2\mu}{6E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2+2\sigma_1\sigma_2 + 2\sigma_2\sigma_3+2\sigma_3\sigma_1\right]$

$= \frac{1}{6E}\begin{bmatrix}2\sigma_1^2+2\sigma_2^2+2\sigma_3^2-2\sigma_1\sigma_2-2\sigma_2\sigma_3-2\sigma_3\sigma_1-6\mu\sigma_1\sigma_2-6\mu\sigma_2\sigma_3-6\mu\sigma_3\sigma_1 \\ +2\mu\sigma_1^2+2\mu\sigma_2^2+2\mu\sigma_3^2+4\mu\sigma_1\sigma_2+4\mu\sigma_2\sigma_3+4\mu\sigma_3\sigma_1\end{bmatrix}$

$= \frac{1}{6E}\left[2(1+\mu)\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right)-2(1+\mu)(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$

$= \frac{(1+\mu)}{3E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1\right]$

$= \frac{(1+\mu)}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$

Distortion energy = $\boldsymbol{\frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}$

For uniaxial loading $\sigma_1 = f_y$, $\sigma_2 = \sigma_3 = 0$

Distortion energy under uniaxial loading = $\frac{1}{12G}\left[2f_y^2\right] = \frac{f_y^2}{6G}$

Distortion energy for uniaxial loading = $\boldsymbol{\frac{f_y^2}{6G}}$

Thus, as per maximum distortion energy theory

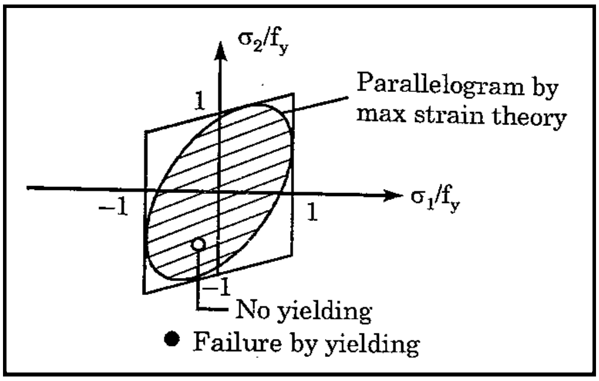
$\Rightarrow \frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le \frac{f_y^2}{6G}$

$\Rightarrow \boldsymbol{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le f_y^2}$

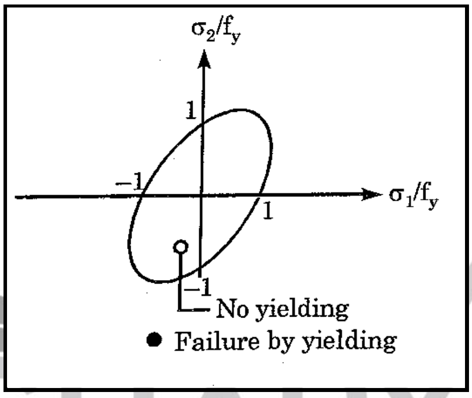
(vi) Octahedral Shear Stress Theory (in 2D) is given by

- $\boldsymbol{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 \le f_y^2}$
- **for design** $\boldsymbol{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 \le \left(\frac{f_y}{F.o.s.}\right)^2}$

4. Maximum Strain Energy Theory



5. Maximum Distortion Energy Theory



Conclusion

(a) Maximum shear stress theory→most conservative

(b) Maximum distortion energy theory → most appropriate for ductile material.

(c) Maximum principal stress theory → most appropriate for brittle material.

All the theories compare the value under general state of stress with that under uniaxial loading. Hence all the theories will give similar result under uniaxial loading (or when one principal stress is large compared to other).

Qu4 **The lists given below refer to a bar of length L, cross sectional area A, Young's modulus E, Poisson's ratio μ and subjected to axial stress 'p' Match List-I with List- II and select the correct answer using the codes given below the lists:**

List-I	List-II
A. Volumetric strain	**1.** $2(1+\mu)$
B. Strain energy per unit volume	**2.** $3(1-2\mu)$
C. Ratio of Young's modulus to bulk modulus	**3.** $\frac{p}{E}(1-2\mu)$
D. Ratio of Young's modulus to modulus of rigidity	**4.** $\frac{p^2}{2E}$
	5. $2(1-\mu)$

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	5	4	1	2
(c)	5	4	2	1
(d)	2	3	1	5

Qu5 **Consider the following statements:**

1. **In a member subjected to uniaxial tensile force the maximum normal stress is the external load divided by the maximum cross-sectional area.**
2. **When the structural member is subjected to uniaxial loading, the shear stress is zero on a plane where the normal stress is maximum.**
3. **In a member subjected to uniaxial loading, the normal stress on the planes of maximum shear stress is less than the maximum.**

Which of these statements are correct?

(a) 1 and 2

(b) 1 and 3

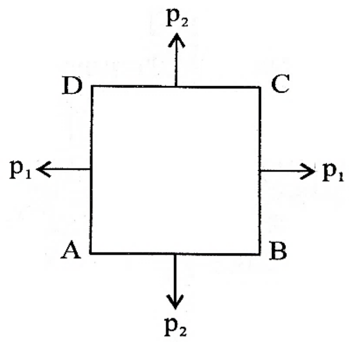
(c) 2 and 3

(d) 1, 2 and 3

Codes:

	A	B	C	D
(a)	5	3	1	4
(b)	5	1	2	4
(c)	3	5	1	2
(d)	3	1	2	5

Qu9 A plane rectangular element is subjected to two normal stresses $'p_1'$ and $'p_2'$ on two mutually perpendicular planes ($p_1 > p_2$) as shown in the figure.



Which one of the following statements is NOT true in this regard?

(a) The planes BC and CD are principal planes.

(b) Shear stress will act on planes inclined to planes AB and CB.

(c) There will not be any normal stress on planes having maximum shear stress.

(d) There will not be any shear stress on planes AB and BC.

Qu12 The principal stresses at a point in a strained material are 'p_1' and 'p_2'. The resultant stress 'p_r' on the plane carrying the maximum shear stress would be

(a) $\frac{(p_1^2+p_2^2)^{1/2}}{2}$

(b) $\left[\frac{p_1^2+p_2^2}{2}\right]^{1/2}$

(c) $\left[2\left(p_1^2+p_2^2\right)\right]^{1/2}$

(d) $2\left[p_1^2+p_2^2\right]^{1/2}$

Answer

1-(b), 2-(a), 3-(d), 4-(a), 5-(c), 6-(d), 7-(b), 8-(a), 9-(c), 10-(b), 11-(d), 12-(b)

Corresponding to these stress elements we can find out maximum principal stress (σ_{max}) and maximum shear stress (τ_{max})

If we have to find out maximum principal stress and maximum shear stress anywhere in the beam, then the points that needs to be investigated are:

(a) For rectangular sections

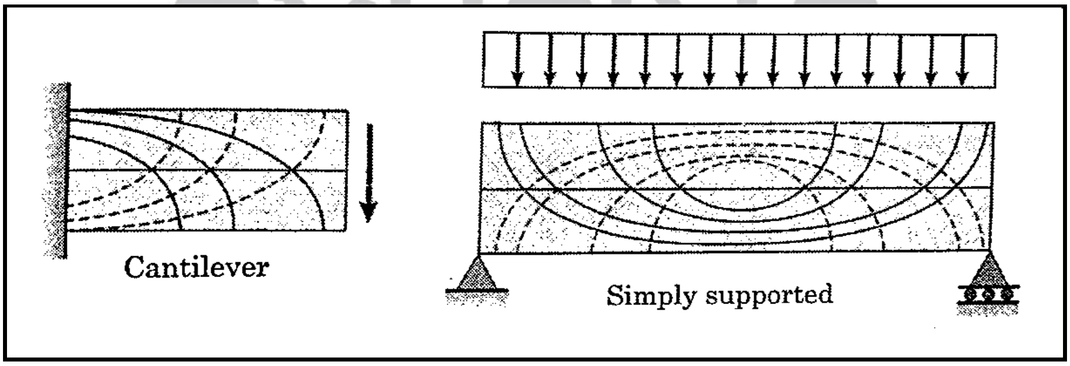
(i) Points of maximum bending stress and point of maximum shear stresses on the section of maximum BM.

i.e., extreme points and neutral axis locations on the section of maximum BM.

(ii) Point of maximum bending stress and point of maximum shear stress on the section of maximum shear force.

(b) For flanged sections, in addition to above points we also need to investigate the junction of web and flange.

STRESS TRAJECTORIES



If every point in a beam is analysed such that we can locate the directions of maximum tensile stress and maximum compressive stress at all points, then we can draw two sets of curves on the beam faces such that tangent to 1st set of curves at any point represents the direction of maximum tensile stress and tangent to other sets of curves at any point represents the direction of maximum compressive stress. These two sets of curves combinedly are called stress trajectories.

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Reinforced Cement Concrete

Education's purpose is to
replace an empty mind with an open one.

Malcolm Forbes

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At Point a

$$\tau_{xz} = \tau_{max} = \frac{Tr_{max}}{J} = \frac{TD/2}{\frac{\pi D^4}{32}} = \frac{16T}{\pi D^3}$$

$$\text{Maximum shearing stress, } \boldsymbol{\tau_{max}} = \frac{\mathbf{16T}}{\boldsymbol{\pi D^3}}$$

$$\sigma_x = \sigma_{max} = \frac{My_{max}}{I} = \frac{M \times D/2}{\frac{\pi D^4}{64}} = \frac{32\,M}{\pi D^3}$$

$$\text{Maximum bending stress, } \boldsymbol{\sigma_{max}} = \frac{\mathbf{32\,M}}{\boldsymbol{\pi D^3}}$$

Note

- If a shaft is simultaneously subjected to a torque T and a bending moment M, then $\frac{\sigma_{max}}{\sigma_{max}} = \frac{2M}{T}$

At Point B

$$\tau_{xy} = -\frac{16T}{\pi D^3}$$

Principal Stresses at A

$$\sigma_{1/2} = \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + (\tau_{max})^2}$$

$$\sigma_{1/2} = \frac{16M}{\pi D^3} \pm \sqrt{\left(\frac{16M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$\boldsymbol{\sigma_{1/2}} = \frac{\mathbf{16}}{\boldsymbol{\pi D^3}}\left[\mathbf{M} \pm \sqrt{\mathbf{M^2 + T^2}}\right]$$

$$\boldsymbol{\tau_{max}} = \frac{\mathbf{16}}{\boldsymbol{\pi D^3}} \sqrt{\mathbf{M^2 + T^2}}$$

τ_{max} occur at 45° to principal stress.

- This is the concept on which equivalent moment M_e and equivalent torque T_c has been defined.

i.e.

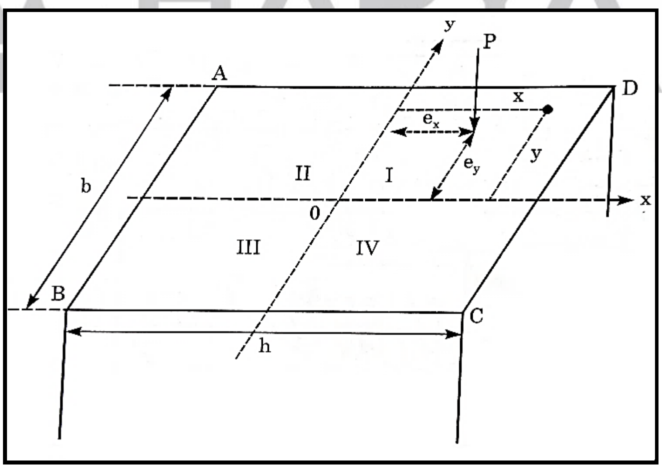
$$\frac{\mathbf{M}_e(\frac{\mathbf{D}}{2})}{\frac{\pi \mathbf{D}^4}{64}} \leq \boldsymbol{\sigma}_{\text{permissible}}$$

$$(\mathbf{T_e\ x\ D/2})\Big/\frac{\pi \mathbf{D}^4}{32} = \boldsymbol{\tau}_{\text{permissible}}$$

- Thus, while designing a section we will ensure that maximum normal stress due to M_e is less than the maximum permissible normal stress and at the same time, maximum shear stress corresponding to T_e must be less then maximum permissible shear stress.

COMBINED BENDING AND AXIAL FORCE

- The figure shown below shows the section of a column in which there is no chance of buckling.
- When there is no chance of buckling, normal stress at any location is given by



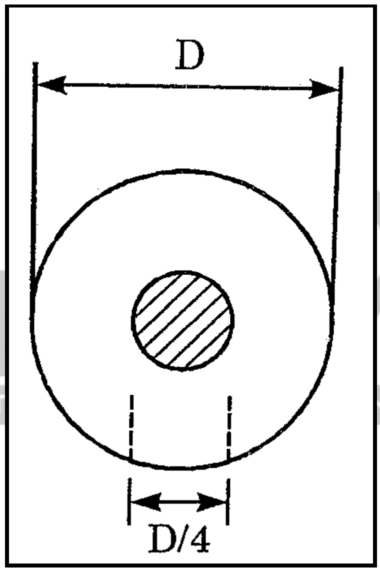
- In the above expression compression is (-) ve and tension is (+) ve
- By putting e_x , x, e_y and y with their algebraic sign the value of σ can be calculated. If σ is (+) ve ⇒normal stress is tensile and if σ comes out to be (-) ve, it means that σ at that point is compressive.

By simple analysis it can be proved that:

- If loading is to the left of line mn, stress at B will be compressive.
- If loading is to the right of mn, stress at B will be tensile.
- mn is the locus of points of application of 'P' for which corner B will have zero stress.
- On similar line, it can be shown that if loading is inside the shaded area, there will be no tension anywhere in the x-section.

This area is called Kern of the section.

Kern of Circular Section



Note

$\Rightarrow \frac{P}{\frac{\pi D^2}{4}} - \frac{\frac{PeD}{2}}{\frac{\pi D^4}{64}} \geq 0$ for no tension any where in the section.

$\Rightarrow \frac{4P}{\pi D^2}\left(1 - \frac{8\,e}{D}\right) \geq 0$ for no tension

$\Rightarrow \frac{8e}{D} \leq 1$

$\Rightarrow e \leq D/8$

$\Rightarrow$ Kern of a circular section will be a circle of diameter $\frac{D}{4}$.

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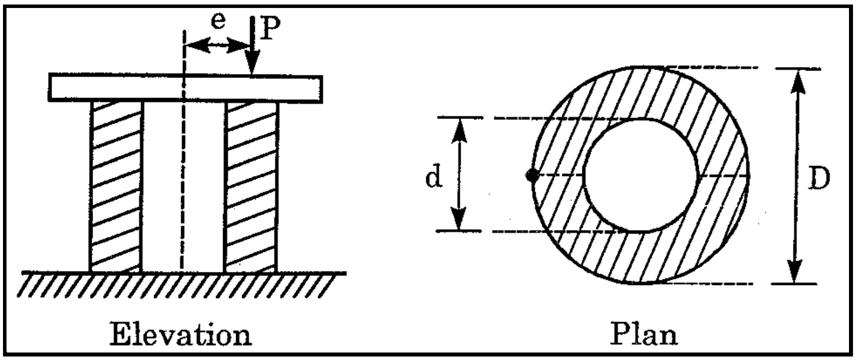
Kern of Hollow Circular Section



For no tension anywhere in the x-section of hollow section.

$$\sigma = \frac{-4P}{\pi(D^2 - d^2)} + \frac{Pe\,D/2}{\frac{\pi}{64}(D^4 - d^4)} \le 0$$

$$\Rightarrow \frac{32Pe}{\pi\frac{(D^4 - d^4)}{D}} \le \frac{4P}{\pi(D^2 - d^2)}$$

$$e \le \frac{4P}{\pi(D^2 - d^2)} x \frac{\pi(D^4 - d^4)}{D\,x\,32P}$$

$$e \le \frac{D^2 + d^2}{8D}$$

$$\textbf{diameter of Kern} = \frac{\mathbf{D^2 + d^2}}{\mathbf{4D}}$$

Kern is circular

Qu3 Which one of the following rules ascertains the maximum permissible eccentricity of loads on circular column so that stresses will always be compressive?

(a) Middle fourth rule

(b) Middle third rule

(c) Middle half rule

(d) Middle two-third rule

Qu4 If ρ is the specific gravity of the material used in the design of a masonry dam of triangular section, then the ratio between the height and base width of the dam for structural safety and stability is equal to

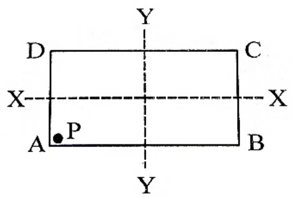
(a) $\sqrt{2\rho}$

(b) $\sqrt{\rho}$

(c) $\frac{1}{\rho}$

(d) $\frac{1}{\sqrt{\rho}}$

Qu5 A reinforced concrete footing loaded with a concentrated load P as shown in the given figure produces maximum bending stresses of 10 kN/m^2 and 15 kN/m^2 due to eccentricities about XX and YY axis respectively. If the direct stress due to load acting at P is 18 kN/m^2 (compressive), then the intensity of resultant stress at corner B will be



(a) 13 kN/m^2 tensile

(b) 13 kN/m^2 compressive

(c) 31 kN/m^2 compressive

(d) 31 kN/m^2 tensile

Qu8 **A Short hollow CI column section A is 150 cm^2 and the section modulus Z= 10×10^5 mm^3 carries**

(i) an axial load of 250 kN, and

(ii) a load of 50 kN on a bracket, the load line being 500 mm from the axis of column

The maximum and minimum stress intensities are

(a) 50 N/ mm^2 tensile and 10 N/mm^2 compressive

(b) 45 N/ mm^2 compressive and 5 N/ mm^2 tensile

(c) 55 N/ mm^2 compressive and 5 N/ mm^2 tensile

(d) 60 N/ mm^2 tensile and 10 N/ mm^2compressive

Qu9 **A section of solid circular shaft with diameter D is subjected to bending moment M and torque T. The expression for maximum principal stress at the section is**

(a) $\frac{2M+T}{\pi D^3}$

(b) $\frac{16\pi}{D^3}(M+\sqrt{M^2+T^2})$

(c) $\frac{16\pi}{D^3}(\sqrt{M^2+T^2})$

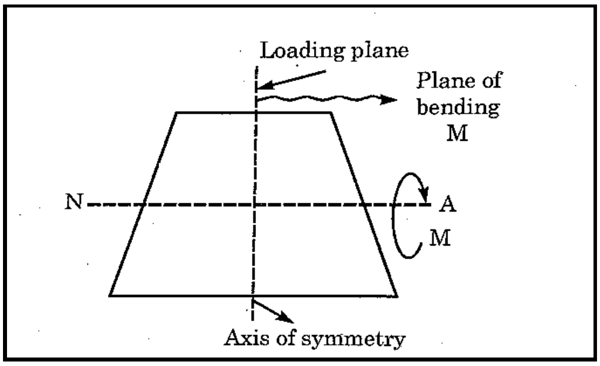
(d) $\frac{16}{\pi D^3}(M+\sqrt{M^2+T^2})$

Answer

1-(b), 2-(b), 3-(a), 4-(b), 5-(b), 6-(a), 7-(d), 8-(b), 9-(d)

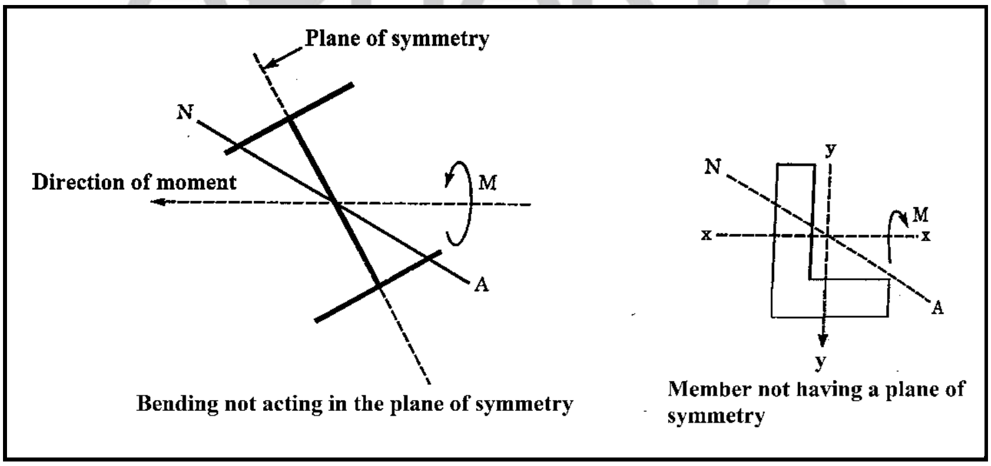
SYMMETRIC BENDING

When member possesses a plane of symmetry and loading (Bending couple) acts in the plane of symmetry, bending is called symmetric bending. In such case, member bends in the plane of couple.



UNSYMMETRIC BENDING

When bending couple does not acts in the plane of symmetry of member either because they act in a different plane or because the member does not possess a plane of symmetry, the bending is called unsymmetric bending. In such case, member does not bend is the plane of couple.



ASSUMPTIONS IN THE THEORY OF PURE BENDING

1. The plane section of the beam before bending remains plane after bending.

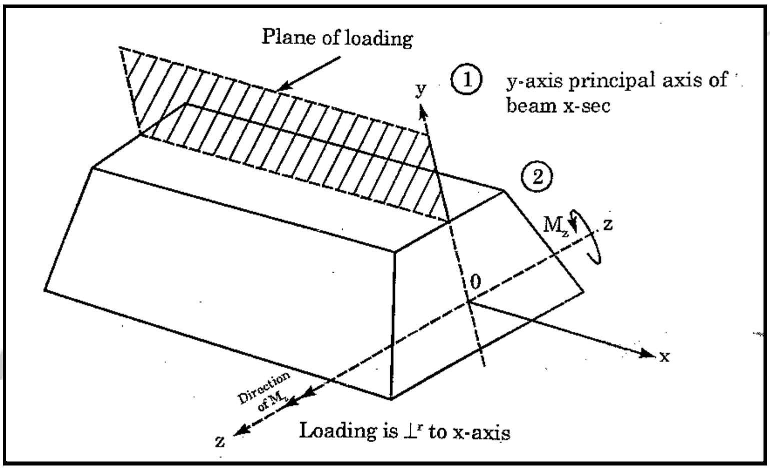
   (i.e., strain variation from neutral axis is linear)

2. The material in the beam is homogeneous, isotropic and obeys Hooke's law

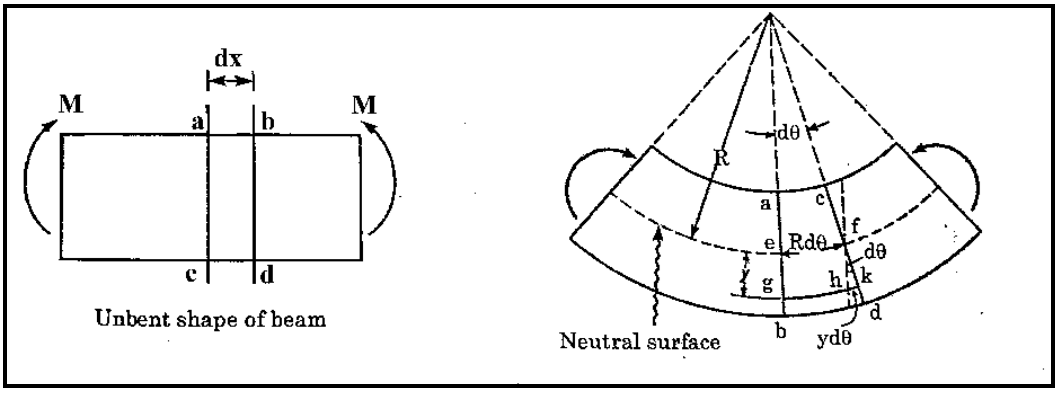
3. Modulus of elasticity in tension and compression are equal.

4. Beam is initially straight and has constant cross-section throughout its length (i.e., beam is prismatic).

5. The plane of loading must contain a principal axis of the beam X section and the loads must be perpendicular to the longitudinal axis of beam.



FLEXURE FORMULA



Hence from (ii) and (iii)

$$\frac{M_z}{I_z} = \frac{\sigma}{y} = \frac{E}{R} \text{ (Flexure formula)-----------(iv)}$$

where, I_z= moment of inertia about C.G. axis (about which bending occurs)

Note that

(−) Hogging moment lead to Tension at Top and Compression at bottom

(+) Sagging moment leads to Tension at bottom and compression at Top

Also, from $\sum F_x = 0$

$\int_A \sigma_x \, dA = 0$

$$\int_A \frac{Ey}{R} dA = 0 \text{ (if Hooke's law is applicable)}$$

$$\frac{E}{R}\int_A y dA = 0 \text{ (where y = distance of any point from N.A.)}$$

$$\frac{E}{R} A\bar{y} = 0 \Rightarrow \bar{y} = 0$$

(where $\bar{y}$ = distance of centroidal axis from N.A.)

Thus, if Hooke's law is applicable, then centroidal axis is the N.A.

$$\sum M_x = \int (\tau_{xy} z - \tau_{xz} y) dA \text{ (Twisting moment)}$$

- For section symmetrical about y-axis and having loading in the symmetrical plane, the element under discussion will have symmetrically placed counterpart so the above integral becomes zero.
- Thus, for symmetrical section having loading in the plane of symmetry, twisting moment= 0. Otherwise, if plane of loading is not symmetrical, the beam will twist.

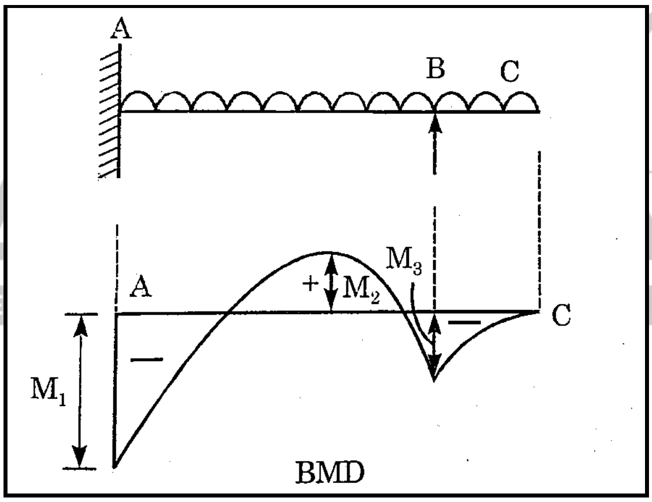
MOMENT OF RESISTANCE (M_R)

Maximum bending moment resisted by a section without undergoing failure is called moment of resistance of the section.

$$\mathbf{M_R = \sigma_{permissible} \times Z}$$

Where $\sigma_{permissible}$ is permissible bending stress. Its value is yields stress/factor of safety i.e. $\frac{\sigma_y}{F.o.s.}.z$

- Thus, larger the value of section modulus, stronger is the beam.
- For designing a beam of uniform section, the section should be selected such that the moment of resistance of the section becomes equal to the applied maximum bending moment.



In the beam shown above, if beam ABC is to be of uniform section, then the relationship

$$\mathbf{Max.\ (M_1, M_2, M_3) = \sigma_{permissible} \cdot z}$$

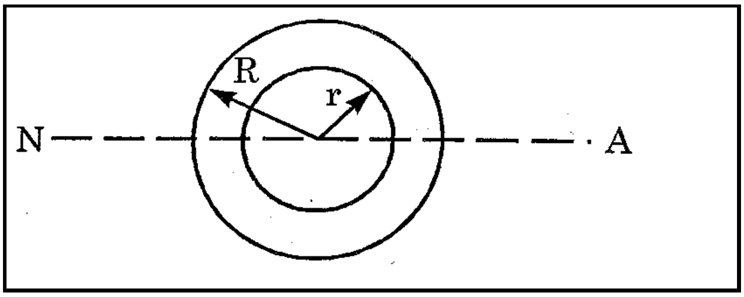
is used to design the section.

If AB and BC can be of different section then the relationship

$$\mathbf{Max.\ (M_1, M_2, M_3) = \sigma_{permissible} \cdot z}$$

$$Z = \frac{\pi D^3}{32}$$

(C) Hollow Circular Section

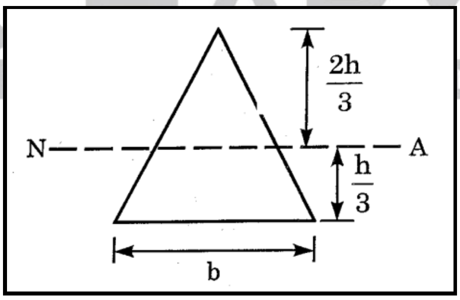


$$I_{N.A.} = \frac{\pi(R^4 - r^4)}{4}$$

$$y_{max} = R$$

$$Z = \frac{\pi(R^4 - r^4)}{4R}$$

(D) Triangular Section



$$I_{N.A.} = \frac{bh^3}{36}, y_{max} = \frac{2h}{3}$$

$$Z = \frac{bh^2}{24}$$

GPSC - CIVIL

Surveying

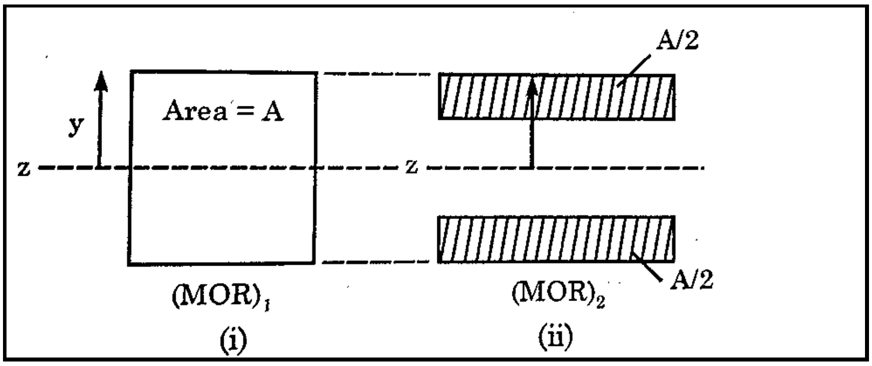
The best Brains of the Nation may be found on the last Benches of the Classroom.

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

JUSTIFICATION FOR USE OF I-SECTION AS A BEAM

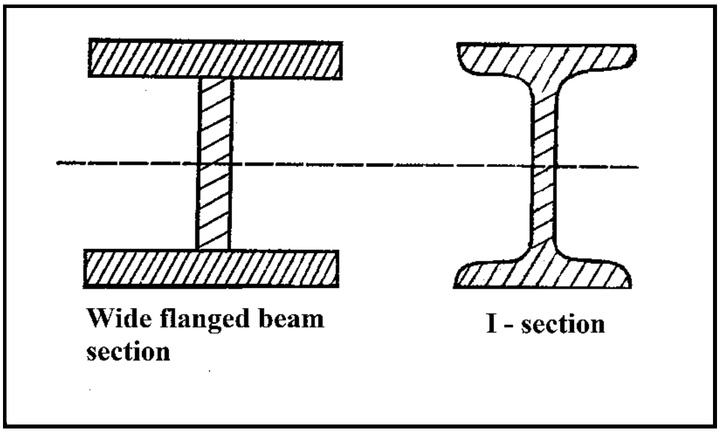
- Moment of resistance of a section is more when Z is more.
- But $Z=\frac{I}{y_{max}}$, Hence for a given y, Z is more if I is more also I is more if more area is located away from NA.



The figure shows that in Fig. (ii) more area is located away from Neutral Axis

$$MOR_2 > MOR_1$$

Hence instead of rectangular section, wide flanged section and I-beams are recommended. In these sections, more than 80% of bending is resisted by flange only. However, very thin web will lead to lateral instability. Hence section should be properly designed.



From the above justification one can easily argue that out of rectangular and circular section of same x- section area rectangular section is more efficient in bending.

BEAM OF CONSTANT STRENGTH OR FULLY STRESSED BEAM

To minimise the quantity of material and thereby to have the lightest possible beam, we can vary the dimensions of cross-section such that max stress at every x-section of the beam is equal to max allowable bending stress in beam. The beam so obtained is called fully stressed beam or a beam of constant strength.

BEAM OF COMPOSITE SECTION

A beam section composed of two different materials is called beam of composite section. The two different materials, can be

(a) Rigidly connected

(b) Simply placed one over the other.

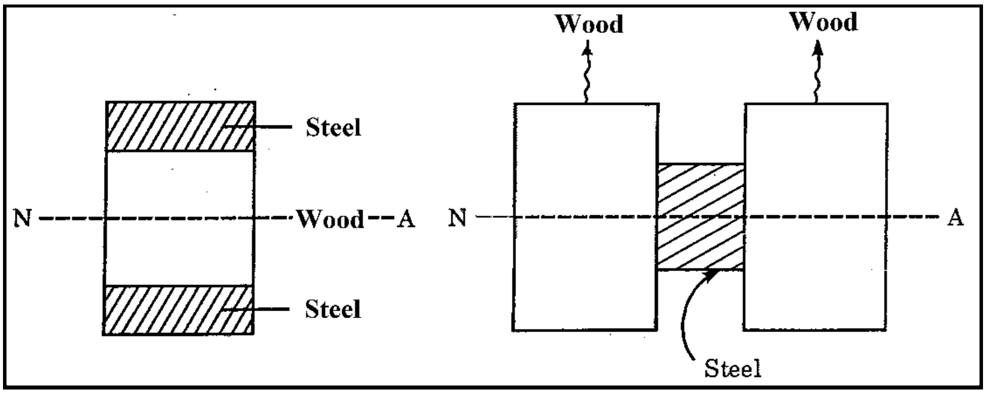
If M is the total BM resisted by the section, then $\mathbf{M = M_1 + M_2}$

Where M_1= BM resisted by 1st material

M_2= BM resisted by 2nd material

Rigidly Connected Composite Section

(i) Symmetrically connected composite section

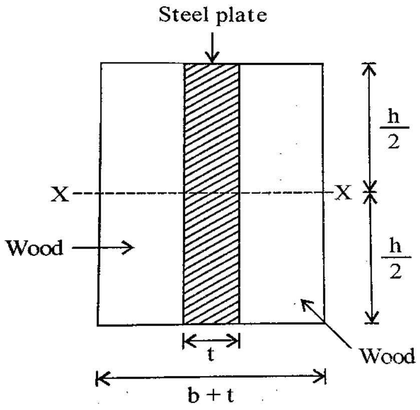


The above beams are also called fliched beams.

For symmetrical section, location of N.A. to easy.

CLEAR YOUR CONCEPT

Qu1 The figure below shown the cross-section of a flitched beam consisting of a steel plate sandwiched between two wooden blocks. The second moment of area of the composite beam about the neutral axis XX is



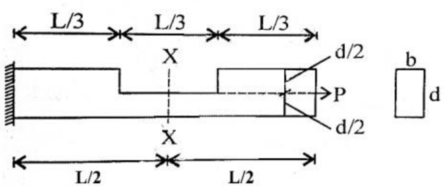
(a) $\frac{bh^3}{12} + \frac{mth^3}{12}$

(b) $\frac{bh^3}{12} + \frac{t(mh)^3}{12}$

(c) $(b + t)\frac{h^3}{12}$

(d) $\frac{bh^3}{12}$

Qu2 The maximum tensile stress at the section X-X shown in the figure below is



(a) $\frac{8P}{bd}$

(b) $\frac{6P}{bd}$

(c) $\frac{4P}{bd}$

(d) $\frac{2P}{bd}$

(a) 1 and 4 only

(b) 2 and 3

(c) 3 and 4

(d) 1, 2 and 4

Qu6 **A steel beam is replaced by a corresponding aluminium beam of same cross-sectional shape and dimensions, and is subjected to same loading. The maximum bending stress will**

(a) Be unaltered

(b) Increase

(c) Decrease

(d) Vary in proportion to their modulus of elasticity

TEST YOUR SELF

Qu7 **A structural beam subjected to sagging bending has a cross-section which is an unsymmetrical I- section. The overall depth of the beam is 300 mm. The flange stresses in the beam are:**

σ_{top} **= 200 N/mm²** → $\sigma_{top} = 200\ N/mm^2$

$\sigma_{bottom} = 50\ N/mm^2$

What is the height in mm of the neutral axis above the bottom flange?

(a) 240 mm

(b) 60 mm

(c) 180 mm

(d) 120 mm

Qu10 A 20 cm long rod of uniform rectangular section, 8 mm wide x 1.2 mm thick is bent into the form of a circular arc resulting in a central displacement of 0.8 cm. Neglecting second-order quantities in computations, what is the longitudinal surface strain (approximate) in the rod?

(a) 7.2 x 10^{-4}

(b) 8.4 x 10^{-4}

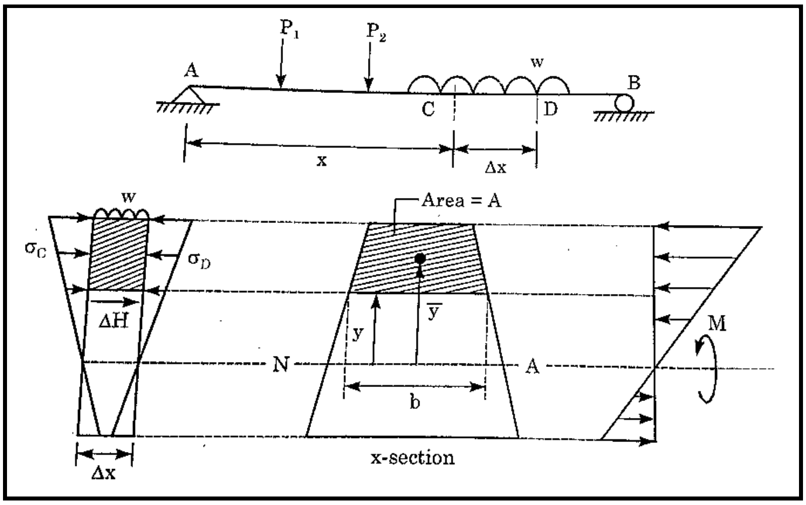
(c) 9.6 x 10^{-4}

(d) 10.8 x 10^{-4}

Answer

1-(a), 2-(a), 3-(a), 4-(d), 5-(d), 6-(a), 7-(b), 8-(c), 9-(d), 10-(c)

SHEAR STRESS IN PRISMATIC BEAM HAVING LOADING IN THE VERTICAL PLANE OF SYMMETRY



$$\sum F_{Horizontal} = 0 \Rightarrow \triangle H + \int_A (\sigma_C - \sigma_D)dA = 0$$

$$\triangle H = \int_A (\sigma_C - \sigma_D)dA = \int_A \frac{(M_D - M_C)y}{I}dA$$

$$= \frac{M_D - M_C}{I}\int_A ydA = \frac{\triangle M}{I}.A\bar{y}$$

$$\triangle H = \frac{\left(\frac{dM}{dx}\times\triangle x\right)A\bar{y}}{I} = \frac{VA\bar{y}}{I}\triangle x$$

$\boldsymbol{\triangle H = \frac{VA\bar{y}}{I}\triangle x}$ = Shear force in length $\triangle x$ of beam

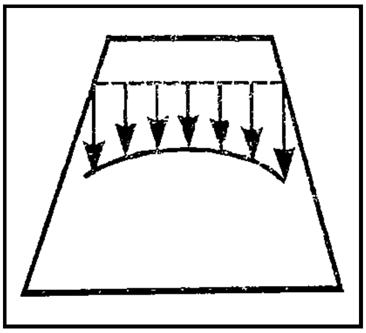
- Shear force unit length of beam = $\frac{\triangle H}{\triangle x} = \frac{VA\bar{y}}{I}$

  Shear force per unit length of beam is called shear flow (q)

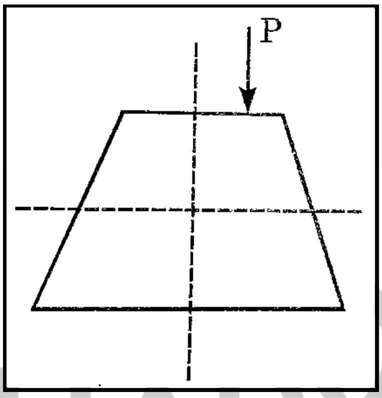
$$\mathbf{q = \frac{VA\bar{y}}{I}}$$

- Shear stress at the level y from N.A. = Complementary shear stress at the level y from N.A.

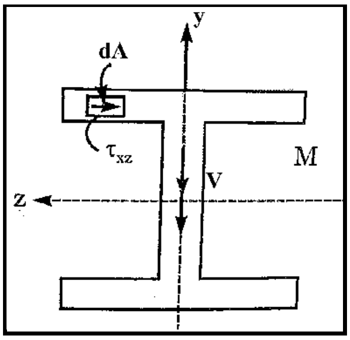
$= \frac{\triangle H}{b\triangle x} = \frac{VA\bar{y}}{Ib}$ where V = SF at the section where shear stress is to be found.



2. If plane of loading does not pass through the symmetrical plane as shown below, twisting of the section will occur. Shear stress $= \frac{VA\bar{y}}{Ib}$ or shear flow $= \frac{VA\bar{y}}{Ib}$ will be applicable only when bending occurs without twisting.



3.



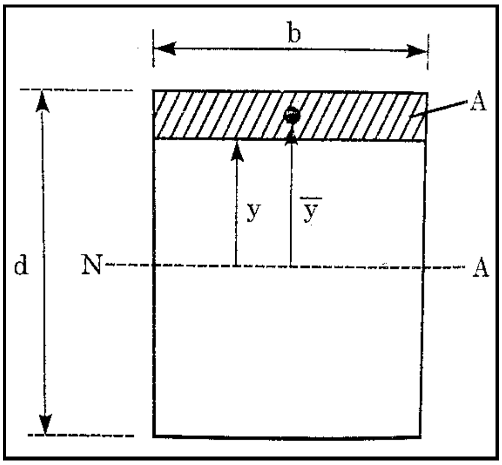
- For the loading shown in the figure, $\sum F_z = 0$

$$\int \tau_{xz} \times dA = 0$$

This does not mean that $\tau_{xz} = 0$. It simply means that $\int \tau_{xz} \times dA = \bar{\tau}_{xz} A = 0$

$\tau_{xz} = 0$ (i.e, average shear stress on x-face in z-direction is zero).

SHEAR STRESS IN RECTANGULAR SECTION



$I = \frac{bd^3}{12}$

$$A\bar{y} = b\left(\frac{d}{2} - y\right)\left[y + \frac{\frac{d}{2}-y}{2}\right] = b\left(\frac{d}{2} - y\right)\left[\frac{y+\frac{d}{2}}{2}\right] = \frac{b\left(\frac{d^2}{4}-y^2\right)}{2}$$

$$\text{Shear stress} = \frac{VA\bar{y}}{Ib} = \frac{Vb\left(\frac{d^2}{4}-y^2\right)}{2 \times \frac{bd^3}{12} \times b}$$

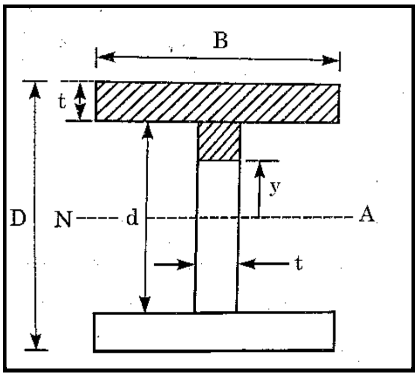
$$\textbf{Shear stress} = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right) = \frac{V}{2I}\left(\frac{d^2}{4} - y^2\right) \text{-------(A)}$$

- Max shear stress occurs at y = 0 (i.e., at neutral axis)
- $(\text{Shear stress})_{max} = \frac{3}{2}\left(\frac{V}{bd}\right) = \frac{3}{2}\ (\text{shear stress})_{av}$
- Variation is parabolic which is as show in the figure below.

Test Series Available..

Total weekly test	: 35
Total mid subject test	: 16
Total full length test	: 13
Mock test	: 16
Total test	: 80

Shear Stress in Web



We know that

$$A\bar{y} = A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + \cdots$$

Hence

$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{d}{2} + \frac{D-d}{4}\right) - t\left(\frac{d}{2} - y\right)\left(y + \frac{\frac{d}{2}-y}{2}\right)$$

$$A\bar{y} = \frac{B(D^2-d^2)}{8} + \frac{t\left(\frac{d^2}{4} - y^2\right)}{2}$$

$$I = \left[\frac{BD^3}{12} - \frac{(B-t)(D-2t)^3}{12}\right]$$

$$= \left[\frac{BD^3}{12} - \frac{(B-t)d^3}{12}\right]$$

$$\tau = \frac{V}{I\,x\,t}\left[\frac{B(D^2-d^2)}{8} + \frac{t\left(\frac{d^2}{4}-y^2\right)}{2}\right] = \text{Shear stress in web}$$

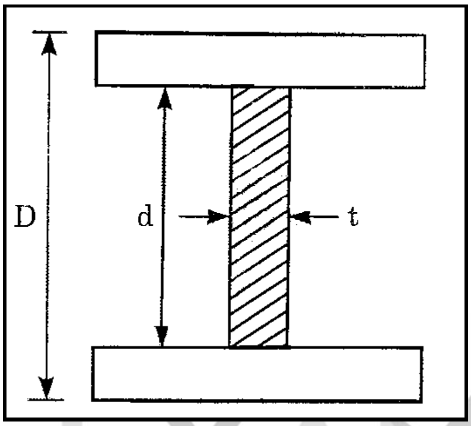
$$\boldsymbol{\tau = \frac{V(D^2-d^2)}{8I}\left(\frac{B}{t}\right) + \frac{V}{2I}\left(\frac{d^2}{4} - y^2\right)} = \text{Shear stress in web}$$

Note:

1. Normally for I -sec, approx. (80-85%) of shear is resisted by web.

2. For approximate calculation of max shear stress in I-sec, following approach is used.

$$\tau_{max} = \frac{V}{d \times t} = \frac{V}{A_{web}}$$



- In design of steel structure maximum permissible value of τ_{max} is 0.45 f_y, where f_y is equal to yield stress.

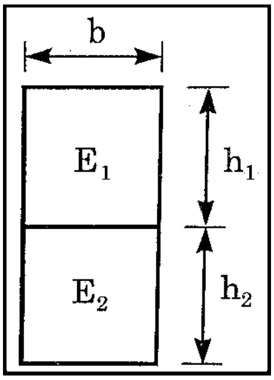
  Thus, we can use $\frac{V}{A_{web}} \leq 0.45f_y$

- However, in actual practice in design, we generally calculate average shear stress average shear stress.

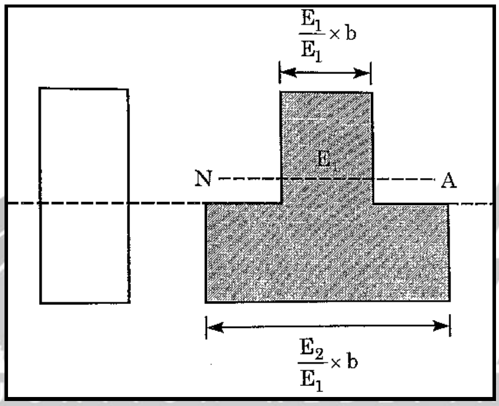
- Average shear stress is given by $\tau_{av} = \frac{V}{Dt}$ and the acceptable limit of τ_{av} in the design of steel structure is 0.4 f_y. Actually, the validity of $\tau_{av} < 0.4f_y$ ensures that $\tau_{max} \leq 0.45f_y$.

COMPOSITE BEAM

We already know that a beam made of two different material is called composite beam. Steps followed in the calculation of shear stress in composite beam are as follows.



1 Draw the transformed section



2. Find N.A. of transformed section.

3. Find shear force per unit length = shear flow = $\frac{VA\bar{y}}{I} = \alpha$

Where

V = Shear force on the section

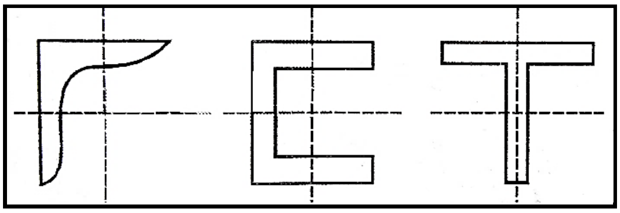
$A\bar{y}$ = Moment of transformed area above the point at which we have to find shear stress about NA

I = Moment of inertia of transformed section about NA

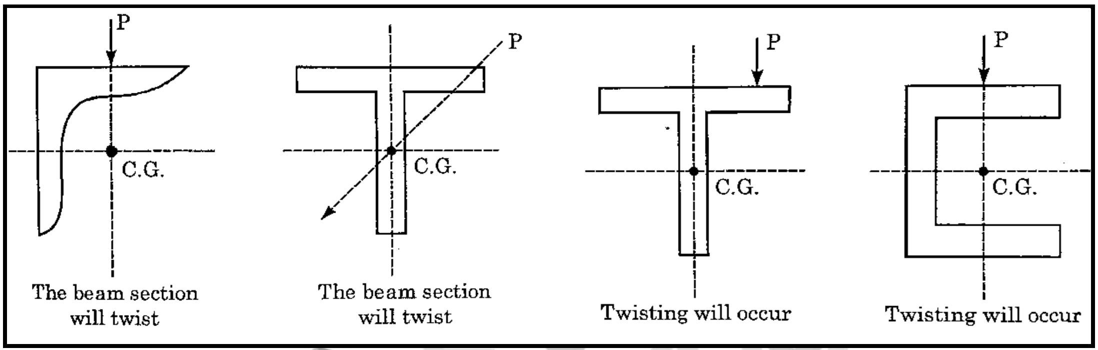
4. Shear stress at any point $= \left(\frac{VA\bar{y}}{I}\right) x \ \frac{1}{b} = \frac{\alpha}{b}$

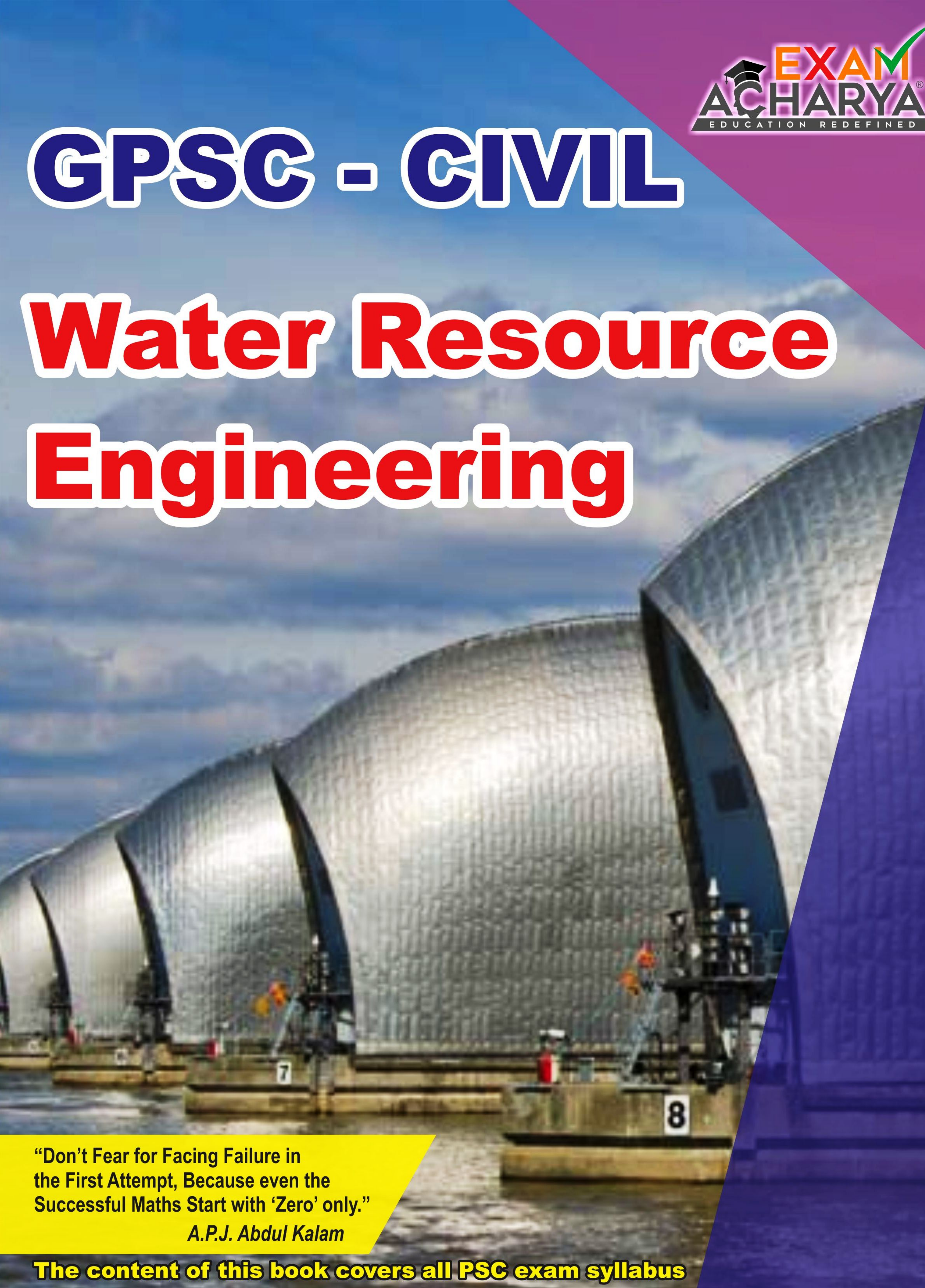
b = actual width at any section.

Hence following sections under pure bending will not twist.



4 If a beam section possesses no plane of symmetry or if it possesses a single plane of symmetry and is subjected to a load that is not contained in the plane of symmetry. (The beam will twist)





EXAM ACHARYA
EDUCATION REDEFINED

GPSC - CIVIL

Water Resource Engineering

"Don't Fear for Facing Failure in the First Attempt, Because even the Successful Maths Start with 'Zero' only."

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

Qu4 **In a beam of solid circular cross-section, what is the ratio of the maximum shear stress to the average shear stress?**

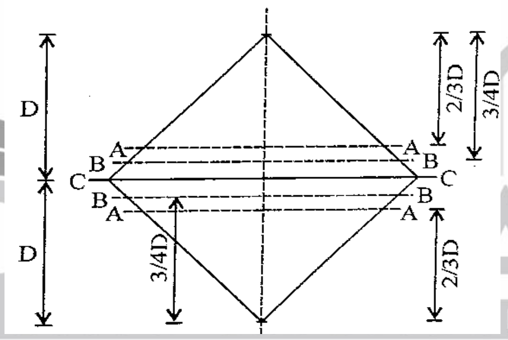
(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{3}{2}$

(d) $\frac{2}{3}$

Qu5 **A beam of square cross-section is placed horizontally with one diagonal horizontal as shown in the figure below. It is subjected to a vertical shear force acting along the depth of the cross section. Maximum shear stress across the depth of cross section occurs at a depth ‘x' from the top. What is the value of x?**



(a) x = 0

(b) x = (2/3) D

(c) x = (3/4) D

(d) x = D

Qu6 **The shear stress at the neutral axis in a beam of triangular section with a base of 40 mm and height 20 mm, subjected to a shear force of 3kN is**

(a) 3 MPa

(b) 6 MPa

(c) 10 MPa

(d) 20 MPa

Qu9 Consider the following statements

1. **In a beam, the maximum shear stress occurs at the neutral axis of the beam cross-section.**
2. **The maximum shear stress in a beam of circular cross-section is 50% more than the average shear stress.**
3. **The maximum shear stress in a beam of triangular cross-section, with its vertex upwards occurs at b/6 above the neutral axis.**

Which of these statements are correct?

(a) 1, 2 and 3

(b) 2 and 3 only

(c) 1 and 2 only

(d) 3 only

Answer

1-(c), 2-(d), 3-(c), 4-(b), 5-(c), 6-(c), 7-(a), 8-(b), 9-(d)

- Torsion produces shearing stress in the section. This can be shown by following figure.

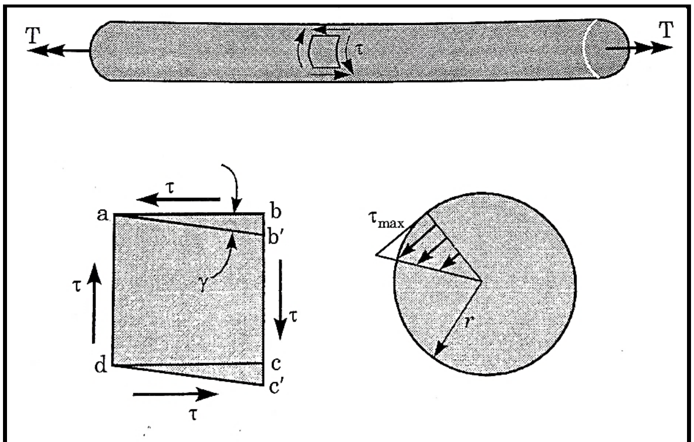
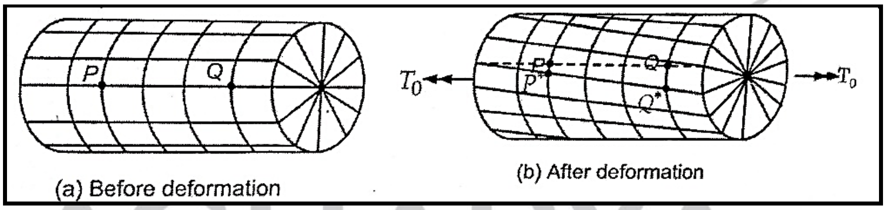


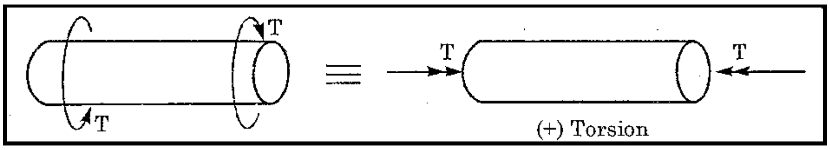
Fig. Shear stresses in a circular bar in torsion.

DEFORMATION UNDER PURE TORSION

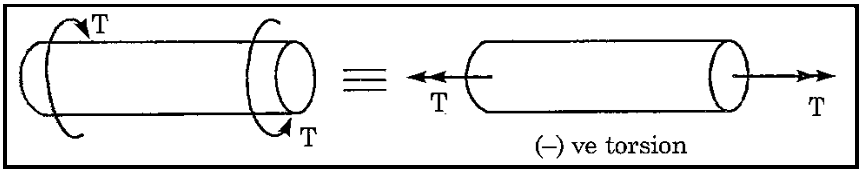


SIGN CONVENTION

- Direction of Torsion Vector is the direction obtained by righthand thumb rule.



- When the direction of torsion vector points towards the section it is taken as (+) ve. Similarly



- When the direction of torsion vector points away from the section it is taken as (-) ve.

From the figure it is clear that the element A before torsion will take a shape as shown in by element B after torsion. As torsion produces shearing strain, shearing strain as shown in element B is γ. Hence yL= r∅

$$\gamma = r\left(\frac{\emptyset}{L}\right)$$

Strain variation is linear over the X-section.

But τ = shear stress = Gγ $=\frac{G\emptyset}{L}$ r [From Hooke's law]

- Shear stress variation is linear over the x-section.
- This linear variation of stress is a consequence of Hooke's law.
- If stress-strain variation is non-linear, the stresses will vary non-linearly

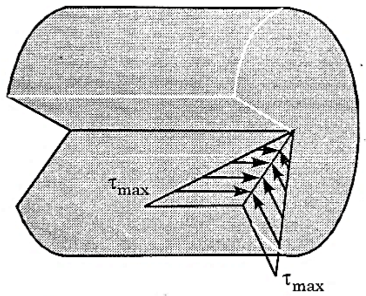
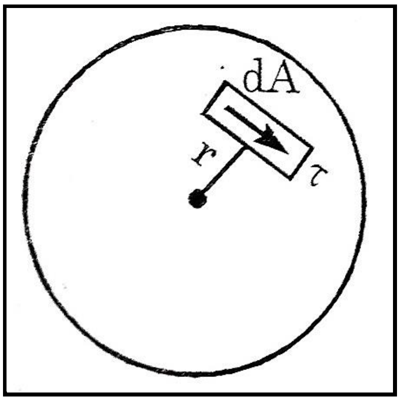


Fig. Longitudinal and transverse shear stresses in a circular bar subjected to torsion.

$$\frac{\tau}{r} = \frac{G\emptyset}{L}$$

Also



$$\int_A (\tau . dA) r = T = \text{Torque}$$

- $\left.\begin{aligned} \sigma_{x'} &= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{x'y'} &= \frac{-(\sigma_x-\sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{aligned}\right]$ [Stress transformation equation]

- At 45° inclined plane i.e. for θ = 45°

  $\sigma_{x'} = 0 + 0 + \tau_{xy}\sin 90° = \tau_{xy}$

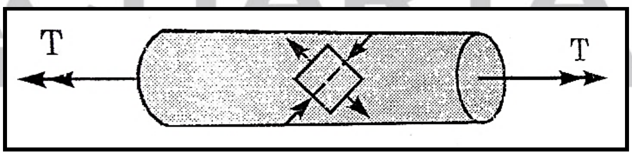
  $\sigma_{y'} = 0 + 0 + \tau_{xy}\sin\ 2\,(\theta + 90°) = \tau_{xy}\sin(180 + 2\theta) = -\,\tau_{xy}$

  $\tau_{x'y'} = 0 + \tau_{xy}\cos 90° = 0$

Max principal stress $(\sigma_1) = \tau_{xy}$(tensile) and min principal stress $(\sigma_2) = -\tau_{xy}$ (compressive)

- Therefore, a rectangular element with side at 45° to the axis of the shaft will be subjected to tensile and compressive stresses.

- If a torsion bar is made of a material that is weaker in tension than in shear, failure will occur in tension along a helix inclined at 45° to the axis. This is the condition for brittle material.



  However, for ductile material, failure is due to shear.

- Thus, Ductile material under torsion will fail on section subjected to largest shear stress i.e. on section $\perp^r$ to the axis of shaft.

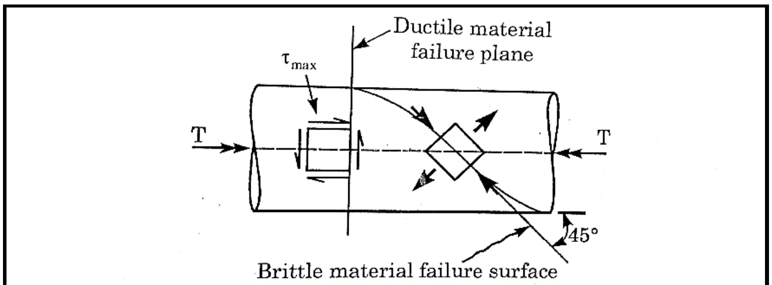
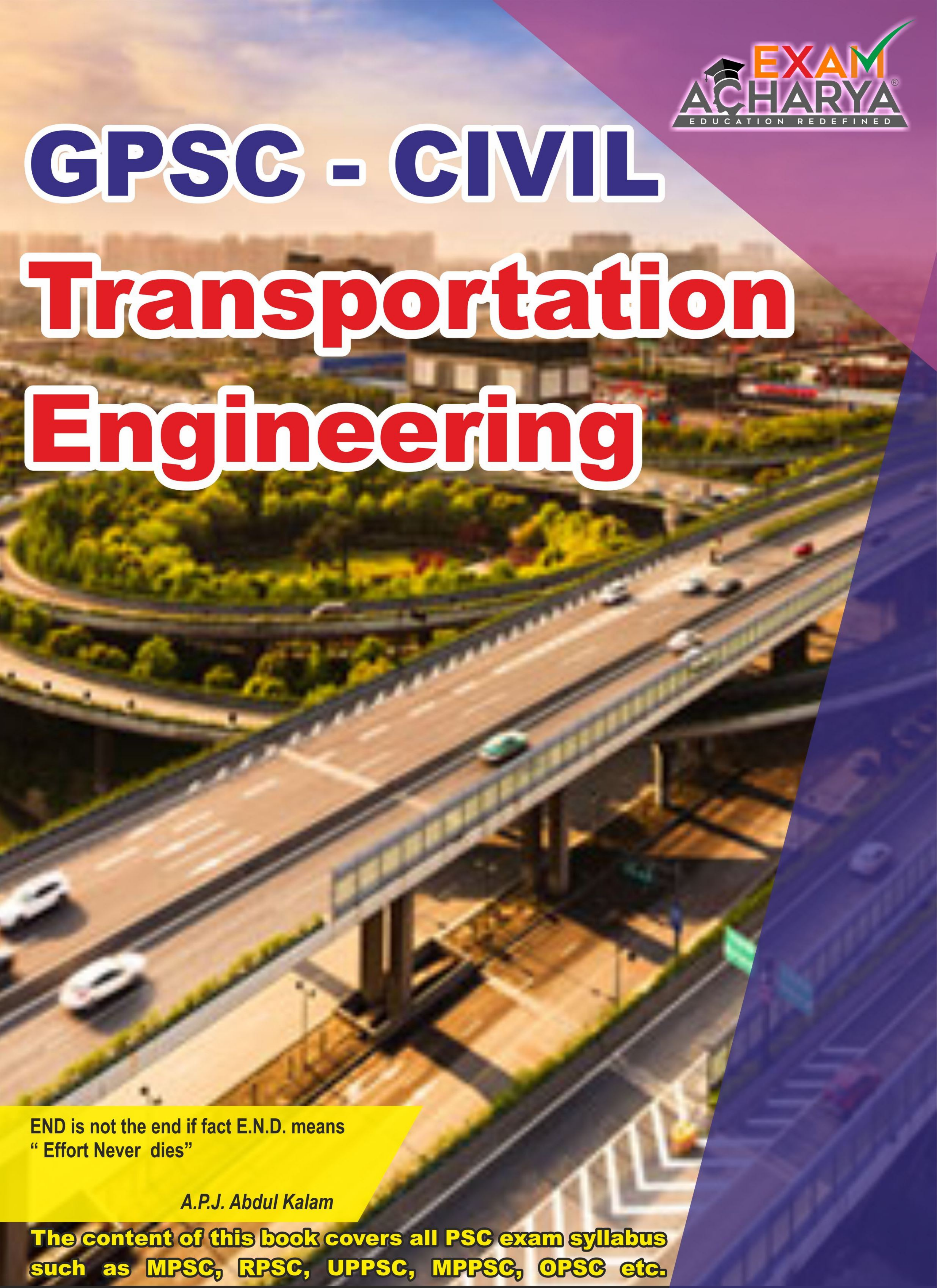


Fig. Potential torsional failure surfaces in ductile and brittle materials.



GPSC - CIVIL

Transportation Engineering

END is not the end if fact E.N.D. means
" Effort Never dies"

A.P.J. Abdul Kalam

The content of this book covers all PSC exam syllabus such as MPSC, RPSC, UPPSC, MPPSC, OPSC etc.

DETERMINATION OF 'G' USING TORSION TEST

- From torsion formula, we know that

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\emptyset}{L}$$

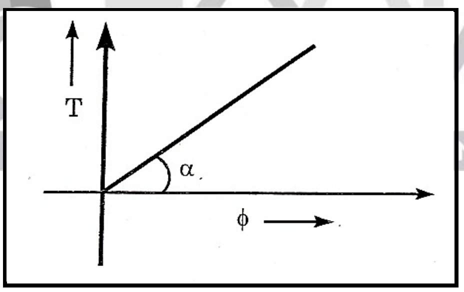
$$\emptyset = \frac{\mathbf{TL}}{\mathbf{GJ}} \text{ Within proportional limit}$$

- In a torsion testing machine, by applying T and measuring $\emptyset$ on a specimen of known length and diameter we can obtain (T-$\emptyset$) plot.

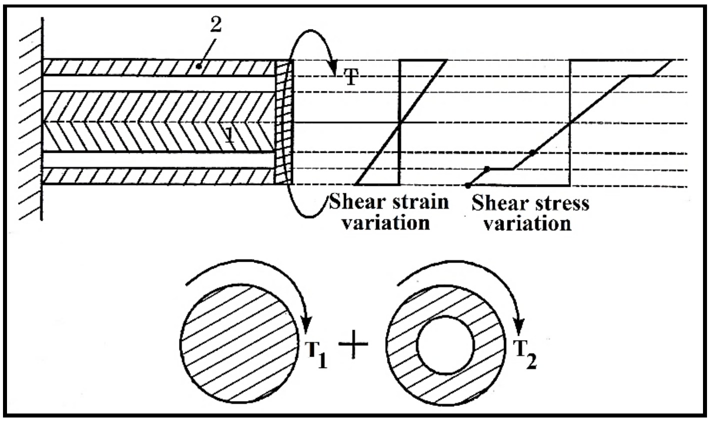
- Slope of this line is $\frac{T}{\emptyset} = \frac{GJ}{L}$

  Hence G = $\left(\frac{T}{\emptyset}\right)\frac{L}{J} = \frac{(\tan\alpha)L}{J}$

- Thus, as length of bar, diameter of bar and slope of (T- $\emptyset$) plot is known, G can be calculated.



(ii) Parallel connection



Let torque resisted by member 1 be T_1 and that resisted by member 2 be T_2.

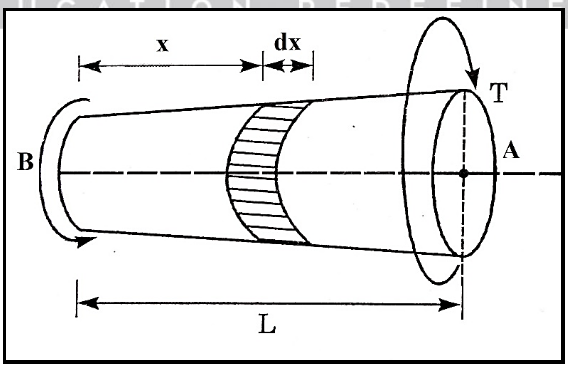
$$T = T_1 + T_2 \text{-------------(i)}$$

Also, $\phi_1 = \phi_2$ (in parallel connection)

$$\frac{T_1 L}{G_1 J_1} = \frac{T_2 L}{G_2 J_2} \text{------------(ii)}$$

By solving (i) and (ii) T_1 and T_2 can be calculated.

ANGLE OF TWIST FOR VARIABLE X-SECTION



$$d\phi = \frac{T.dx}{GJ}$$

$$\phi_{AB} = \int_0^L \frac{Tdx}{GJ}$$

Where

P = Power transmitted

w = Rotational speed in rad/sec

w = 2πf, where f = frequency in Hz

w = $\frac{2\pi N}{60}$, where N = speed in rpm

Note

➢ We can design the diameter of the shaft based on the requirement of power transmission. For design of shaft

$\frac{T_r}{J} \leq \tau_{per}$

$\frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} \leq \tau_{per}$

Find the diameter

THIN-WALLED HOLLOW SHAFT

For thin-walled section, max shear stress is taken corresponding to mean radius.

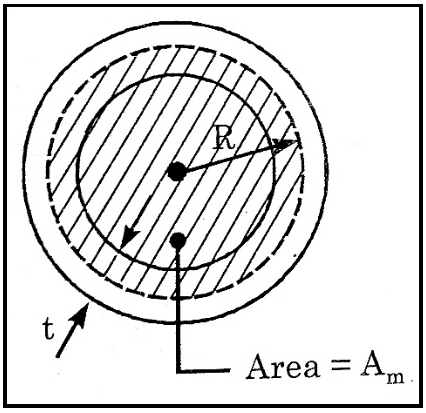
$$J = (2\pi Rt)\,R^2$$

$$J = 2\pi R^3 t$$

$$\tau_{max} = \frac{TR}{J} = \frac{T}{2\pi R^2 t}$$

$$(\tau_{max}.t) = \frac{T}{2\pi R^2}$$

$$\boldsymbol{\tau.t = \frac{T}{2.A_m}} \text{ -----------(i)}$$



CLEAR YOUR CONCEPT

Qu1 Consider the following statements:

If a solid circular shaft and a hollow circular shaft have the same torsional strength, then

1. **The weight of the hollow shaft will be less than that of the solid shaft**
2. **The external diameter of the hollow shaft will be greater than that of the solid shaft**
3. **The stiffness of the hollow shaft will be equal to that of the solid shaft**

Which of these statements is/are correct?

(a) 1, 2 and 3

(b) 2 and 3

(c) 1 alone

(d) 1 and 2

Qu2 A shaft turns at 150 rpm under a torque of 1500 Nm. Power transmitted is

(a) 15π kW

(b) 10π kW

(c) 7.5π kW

(d) 5π kW

TEST YOUR SELF

Qu6 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
A. Torque-twist relationship for a circular shaft	1. $\frac{1}{2}\sqrt{\sigma^2 + 4\lambda^2}$
B. Strain energy of elastic torsion	2. $\frac{Gr\theta}{L}$
C. Circumferential shear stress	3. $\left(\frac{GJ}{2L}\right)\theta^2$
D. Maximum shearing stress due to combined torsion and direct stress	4. $\frac{GJ}{L}\theta$

Codes

	A	B	C	D
(a)	2	3	4	1
(b)	4	1	2	3
(c)	2	1	4	3
(d)	4	3	2	1

Qu7 Two shafts having same length and material are joined in series and subjected to a torque of 10 kN-m. If the ratio of their diameters is 2: 1, then the ratio of their angles of twist is

(a) 16: 1

(b) 2:1

(c) 1: 2

(d) 1: 16